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Volume XL Number 1

Edited by William David Reeve

Signed Areas Applied to "Recreations of Geometry"

By H. C. Trimble

Iowa State Teachers College, Cedar Falls, Ia.

The teacher of plane geometry is sometimes confronted with "proofs" which seem to contradict the postulates upon which the subject rests. Eight such proofs are listed in Wentworth's Plane Geometry, Revised by Wentworth and Smith, Ginn and Company, 1910, pages 273 to 276, under the heading "Recreations of Geometry." Variations of these problems continue to arise and to perplex perseverent students of plane geometry.

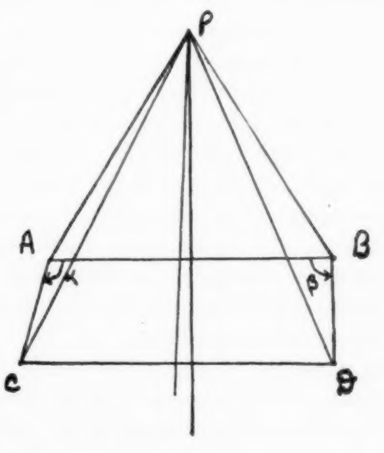
In geometry as in algebra some of the apparent contradictions arise out of an inadvertent division by zero. For example, in the fourth problem listed in Wentworth it is "proved" that part of a line equals the whole line. This is achieved by dividing both members of an equation by zero.

Others of the problems introduce more or less subtle errors in applying geometric theorems.

It is the third class of problems, which depend upon errors in drawing the figure, which will be discussed below. It is felt that these problems have more than recreational value. They illustrate some of the limitations of the geometry of Euclid upon which courses in plane geometry are based. Four of the eight problems listed by Wentworth are of this third class.

The way in which my interest in "proofs" of contradictory propositions was recently aroused may remind the reader of experiences of his own. Just a week before the close of the summer term one of our mathematics majors came in on a Friday morning looking a bit the worse for wear. He had been up most of the night staring at a figure which seemed to prove that two angles which had been drawn unequal must necessarily be equal. Otherwise two triangles having three sides of one equal respectively to three sides of the other were not congruent. A truly bad situation!

The figure which he drew for me and his explanation of the figure appear below:



AB is any line segment.

At A and B construct angles α and β respectively, $\alpha \neq \beta$, $\alpha \neq 180-\beta$.

Make $AC=BD$, and draw CD.

Clearly CD is not parallel to AB.

Draw the right bisectors of AB and CD. Since AB is not parallel to CD, these right bisectors meet at some point P. Draw PA, PB, PC, and PD.

In triangles PAC and PBD.

$AC=BD$ by construction.

$AP=BP$ since P is on the right bisector of AB.

$CP=DP$ since P is on the right bisector of CD.

Hence the triangles are congruent and angle PAC = angle PBD.

But in the isosceles triangle PAB, angle PAB = angle PBA.

Hence angle α = angle β by subtracting equals from equals.

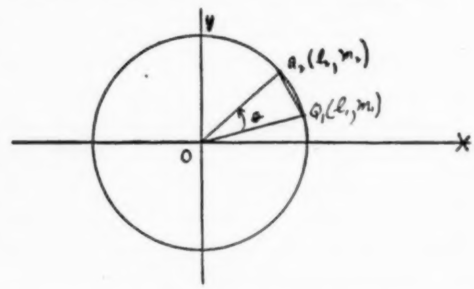
The fact that no contradiction arose in some of the figures which the student had drawn should perhaps have made the difficulty immediately clear. When AC and BD were not nearly parallel one the triangles PAC and PBD "turned inside out." However the student was convinced on the basis of rather carefully drawn figures that over a considerable range of positions of AC and BD the contradiction would surely arise.

The difficulty of locating the point P accurately as the point of intersection of two nearly parallel lines was not appreciated by the student. Even after some careful drawing it was by no means apparent that the point P must recede sufficiently from AB to make either triangle PAC or triangle PBD turn inside out and thus make either angle α or angle β exterior to the corresponding triangle. The argument that such a condition must hold because otherwise a contradiction would arise was a wholly unsatisfactory one at this stage in the student's thinking.

An attempt to apply the conventional methods of analytic geometry to establish the orientation of triangle PAC with respect to triangle PBD led to a rather difficult division of the problem into several different cases. For example when the slopes of the lines PB and PD were compared it was essential to know whether P was above or below AB.

An analysis of the problem in terms of the signed areas of the two triangles was finally made. The theory upon which this analysis rests is presented in an elementary way in "Analytic Geometry" by Francis D. Murnaghan, Prentice Hall, 1946.

The approach to analytic geometry through the study of vectors which is employed in this text provides a number of methods of attack which have previously been available only to students of higher mathematics. It is hoped that the application of one of these tools to the present problem will excite the interest of some readers and lead them to study what Murnaghan calls the alternating product of two plane vectors. The following intro-



duction to signed areas of triangles is couched in the familiar language of conventional analytic geometry for the benefit of the more casual reader. Consider two points $Q_1(l_1, m_1)$ and $Q_2(l_2, m_2)$ on the unit circle. Clearly

$$l_1^2+m_1^2=1$$

$$l_2^2+m_2^2=1.$$

Also by applying the law of cosines to the triangle Q_1OQ_2

$$\overline{Q_1Q_2}^2=\overline{OQ_1}^2+\overline{OQ_2}^2-2\overline{OQ_1}\ \overline{OQ_2}\cos\theta.$$

That is,

$$(l_2-l_1)^2+(m_2-m_1)^2=2-2\cos\theta.$$

Hence

$$(l_1^2+m_1^2)+(l_2^2+m_2^2)-2(l_1l_2+m_1m_2)=2-2\cos\theta$$

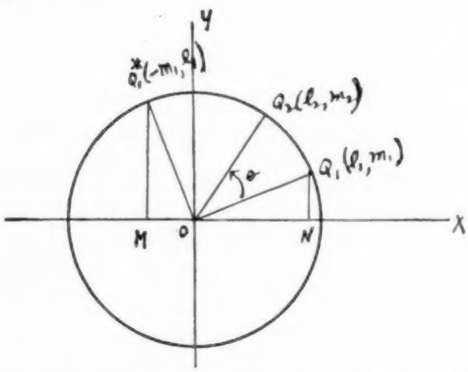
or

$$2-2(l_1l_2+m_1m_2)=2-2\cos\theta$$

and

$$\cos\theta = l_1 l_2 + m_1 m_2.$$

This is the familiar cosine formula which is sometimes overlooked in elementary plane analytic geometry, or at least is introduced only as a preparation for the study of its analogue in solid analytic geometry.



If now a third point Q_1^* is placed on the unit circle 90° ahead of Q_1, that is, such that angle $Q_1OQ_1^* = 90°$, it is evident that the coordinates of Q_1^* are $(-m_1, l_1)$. This follows from the fact that triangle OQ_1N is congruent to triangle Q_1^*OM.

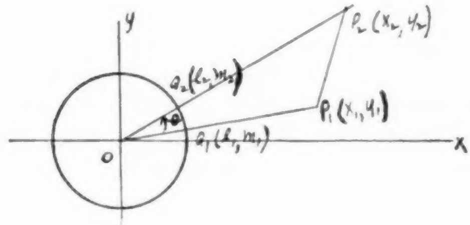
By the cosine formula

$$\cos(Q_2OQ_1^*) = -l_2 m_1 + l_1 m_2.$$

But since $\cos(Q_2OQ_1^*) = \sin\theta$, θ being complementary to $Q_2OQ_1^*$, we have

$$\sin\theta = -l_2 m_1 + l_1 m_2.$$

Now let $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ be any two points in the plane.



Also let $OP_1 = r_1$, $OP_2 = r_2$, angle $P_1OP_2 = \theta$.

The area of triangle P_1OP_2 is

$$K = \tfrac{1}{2} r_1 r_2 \sin\theta.$$

Clearly $K > 0$ when $0 < \theta < 180°$

$K < 0$ when $180° < \theta < 360°$

If $\theta = 0°$, or $\theta = 180°$, the triangle P_1OP_2 is degenerate and $K = 0$.

For the purposes of the ensuing argument, we shall say that $K > 0$ if and only if P_2 lies on the left as one goes along the line OP_1 from O to P_1, and $K < 0$ if and only if P_2 lies on the right as one goes along the line OP_1 from O to P_1.

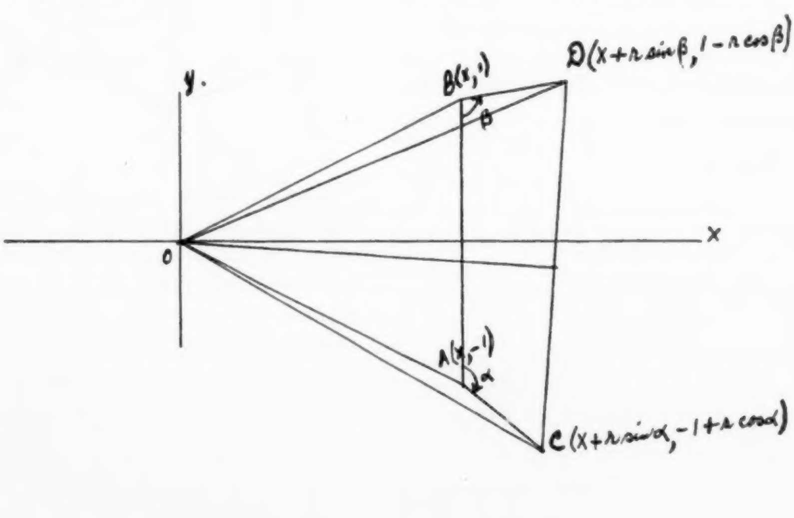
Note that $x_1 = r_1 l_1$, $y_1 = r_1 m_1$, $x_2 = r_2 l_2$, $y_2 = r_2 m_2$. Hence

$$r_1 r_2 \sin\theta = -r_1 r_2 l_2 m_1 + r_1 r_2 l_1 m_2$$
$$= -x_2 y_1 + x_1 y_2$$

and

$$K = \tfrac{1}{2}(-x_2 y_1 + x_1 y_2),$$

This is the formula for the signed area of triangle P_1OP_2 which is to be used in discussing the orientations of the two triangles of the original problem.



In the figure the origin O replaces the intersection P of the perpendicular bisectors of AB and CD. The x-axis is taken as the perpendicular bisector of AB, and the ordinate of B is taken as the unit of measurement. Hence if B is the point $(x, 1)$ A is the point $(x, -1)$. Also C has coordinates $(x+r\sin\alpha,\ -1+r\cos\alpha)$ and D has coordinates $(x+r\sin\beta,\ 1-r\cos\beta)$, where $AC=BD=r$.

Since O lies on the perpendicular bisector of CD, $OD=OC$; that is

$$(x+r\sin\beta)^2+(1-r\cos\beta)^2 = (x+r\sin\alpha)^2+(-1+r\cos\alpha)^2.$$

Hence

$$x^2+2xr\sin\beta+r^2\sin^2\beta+1-2r\cos\beta+r^2\cos^2\beta = x^2+2xr\sin\alpha+r^2\sin^2\alpha+1-2r\cos\alpha+r^2\cos^2\alpha$$

and

$$2r(x\sin\beta-\cos\beta)+r^2(\sin^2\beta+\cos^2\beta) = 2r(x\sin\alpha-\cos\alpha)+r^2(\sin^2\alpha+\cos^2\alpha)$$

from which

$$x\sin\beta-\cos\beta=x\sin\alpha-\cos\alpha$$

since

$$\sin^2\beta+\cos^2\beta=\sin^2\alpha+\cos^2\alpha=1$$

and $r\neq 0$.

Hence

$$x=\frac{\cos\alpha-\cos\beta}{\sin\alpha-\sin\beta} \quad \text{since} \quad \begin{array}{l}\alpha\neq\beta\\ \alpha\neq 180-\beta.\end{array}$$

Twice the signed area of triangle OBD is

$$-(x+r\sin\beta)+x(1-r\cos\beta)$$
$$=-r(\sin\beta+x\cos\beta)$$
$$=-r\left(\sin\beta+\frac{\cos\alpha-\cos\beta}{\sin\alpha-\sin\beta}\cos\beta\right)$$
$$=-r\left(\frac{\sin\alpha\sin\beta-\sin^2\beta+\cos\alpha\cos\beta-\cos^2\beta}{\sin\alpha-\sin\beta}\right)$$
$$=-r\left(\frac{\sin\alpha\sin\beta+\cos\alpha\cos\beta-1}{\sin\alpha-\sin\beta}\right).$$

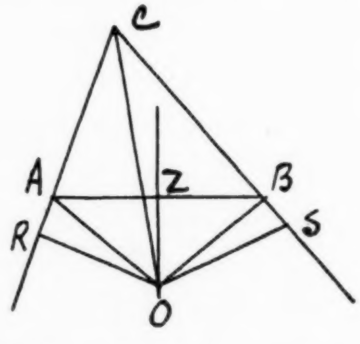
Twice the signed area of triangle OAC is

$$-(x+r\sin\alpha)(-1)+x(-1+r\cos\alpha)$$
$$=r(\sin\alpha+x\cos\alpha)$$
$$=r\left(\sin\alpha+\frac{\cos\alpha-\cos\beta}{\sin\alpha-\sin\beta}\cos\alpha\right)$$
$$=r\left(\frac{\sin^2\alpha-\sin\alpha\sin\beta+\cos^2\alpha-\cos\alpha\cos\beta}{\sin\alpha-\sin\beta}\right)$$
$$=-r\left(\frac{\sin\alpha\sin\beta+\cos\alpha\cos\beta-1}{\sin\alpha-\sin\beta}\right)$$

= twice the signed area of triangle OBD.

Now as one goes along OB, D lies either on the right or on the left. Suppose D lies on the right so that the signed area of triangle OBD is negative. Then the signed area of triangle OAC is negative and C lies on the right as one goes along OA. Similarly if D lies on the left of OB, C lies on the left of OA. This proves that either angle α is exterior to triangle OAC or else angle β is exterior to triangle OBD. Hence the "contradiction" in the original proof was only apparent.

The method of signed areas will be applied to one more problem. This is the first of the recreations listed in Wentworth, the "proof" that every triangle is isosceles.



Triangle ABC is assumed not isosceles.

The right bisector OZ of AB intersects the bisector CO of angle ACB at O.

Draw OR and OS perpendicular to AC and CB respectively.

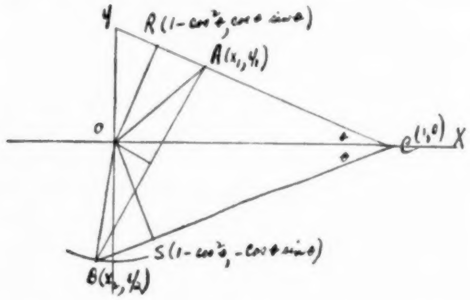
Join OA and OB.

It follows that the triangles OAR and OBS are congruent and hence $AR=BS$.

Also the triangles ORC and OSC are congruent and hence $CR=CS$.

Hence by subtracting equals from equals, $AC=CB$ and triangle ABC is isosceles.

Again the error is in the figure. The points R and S will be proved neither both external nor both internal points of CA and CB. Hence AC and BC are neither the differences of equals nor the sums of equals.



For the purposes of the analysis, OC is taken along the x axis and the unit of measurement is chosen as the distance OC.

In the figure $CR=CS=\cos\theta$, and R and S have the coordinates indicated.

If A has coordinates (x_1, y_1) it follows that

$$\frac{y_1}{\cos\theta\sin\theta}=\frac{1-x_1}{\cos^2\theta}$$

so that

$$\frac{y_1}{\sin\theta}=\frac{1-x_1}{\cos\theta}.$$

If B has coordinates (x_2, y_2) it follows similarly that

$$\frac{y_2}{-\sin\theta}=\frac{1-x_2}{\cos\theta}.$$

Since $OA=OB$, $x_1^2+y_1^2=x_2^2+y_2^2$, and hence

$$x_1^2+\frac{\sin^2\theta}{\cos^2\theta}(1-x_1)^2=x_2^2+\frac{\sin^2\theta}{\cos^2\theta}(1-x_2)^2.$$

Upon simplification

$$x_1^2-2x_1\sin^2\theta=x_2^2-2x_2\sin^2\theta.$$

Adding $\sin^4\theta$ to both members and extracting the square root

$$x_1-\sin^2\theta=\pm(x_2-\sin^2\theta).$$

Hence $x_1=x_2$ which clearly makes triangle ABC isosceles, or

$$x_1=-x_2+2\sin^2\theta.$$

Hence by the assumption that triangle ABC is not isosceles

$$x_1=-x_2+2\sin^2\theta.$$

Twice the signed area of triangle OAR is

$$_1(1-\cos^2\theta)+x_1\cos\theta\sin\theta$$

$$=-\frac{\sin\theta}{\cos\theta}(1-x_1)\sin^2\theta+x_1\cos\theta\sin\theta$$

$$=\tan\theta(x_1-\sin^2\theta)=\tan\theta(-x_2+\sin^2\theta).$$

Twice the signed area of triangle OBS is

$$-y_2(1-\cos^2\theta)-x_2\cos\theta\sin\theta$$

$$=\tan\theta(-x_2+\sin^2\theta)$$

$=$ twice the signed area of triangle OAR.

Hence if R is on the left as one proceeds from 0 to A, S is on the left as one proceeds from O to B. In other words, if R is exterior to CA then S is interior to CB. Similarly if R is interior to CA then S is exterior to CB.

The two examples discussed above illustrated the dependence of Euclid's geometry upon carefully drawn figures which avoid special placement of the lines near critical positions. The method of signed areas, based entirely upon elementary concepts, provided a complete explanation of the manner in which the apparent contradictions arose.

The Problem of a Non-College Preparatory Curriculum in Mathematics and Suggestions for Its Solution

By C. C. Fabing
Susan Miller Dorsey High School, Los Angeles, Calif.

Most mathematics teachers have been aware that the non-college student has been neglected so far as getting his rightful share of mathematics has been concerned. In most instances the awareness was passed passed off with a shrug of the shoulders and the blame for not establishing these courses, passed onto some one else. The need has become so acute that the tail is wagging the dog and it would seem that some action may be taken to aid in solving the problem. If you will check the number of graduates in your high school and the number of these graduates who enter college, you will find that only about 15% of the group go on to college. It must be admitted then, that we have constructed our mathematics curriculum largely for the 15% and the remaining 85% have received little or no consideration in mathematics and most of that was a make shift, hit or miss proposition. In some high schools, it is possible for a pupil to graduate without being required to pursue any class in mathematics. If a search were made, I suspect that many more high schools are permitting this situation than we know. Mathematics teachers as a group are rather complacent and hold the dignity of their profession on a high plane. This is as it should be. Since we are supposed to know the meaning of facts as expressed in figures, then we must admit that 85% of a graduating class is a greater responsibility than 15% of the class. To reach 85% of the class, I hope that we will bestir ourselves from this smug complacency in academic mathematics and lend a helping hand to those who need, but seldom get any mathematics. We must become mathematical missionaries and carry the gospel truth to the majority who need aid in mathematics other than that prescribed for the selected few who go to college. We must show the way or continue to see school subjects with less concrete usefulness and more aggressive leaders reducing our mathematics time in the curriculum.

First, there has never been a name selected for the non-college preparatory mathematics classes that has an ear appeal to the prospective pupil. Most of the names thus far have a stigma in the title. It blares out, that he who registers in this course, has been classified as one below average intelligence. It creates in the mind and fosters the belief in the mind of the pupil that he is below par mentally and the doors to his future education have been closed. The fact remains that sometimes, he is below average in intelligence, but let's not accenturate the fact. Let's permit him to use what mental ability he has to its full. Pupils who register in the non-college preparatory mathematics classes are often known as dumb-bells, dim-wits, beetle brains or some other nom de plume that immediately sets him up as a mental and social outcast in his immediate school society. Sometimes those who are most arrogant in this name calling are those in the academic mathematics classes who are below the median in the academic mathematics classes and could profit by being in a non-college preparatory mathematics class. Shop Mathematics, Applied Mathematics, Economic Mathematics etc. are titles which are obscure and yet intimate that they are for those who have not received their full measure of intelligence. Why not use a title which has a modern appeal to those who are prospective members for the classes such as, The Airway Through Mathematics, The Petty Figure Course, The Social Prestige of Figures, Mathematics Appreciation, or Boogie Woogie Estimations. Our co-workers in other fields of the curriculum have realized the value of a name as an enticement to pupils and have capitalized on it.

There is a committee working in a large city in California on non-college preparatory mathematics that have hit upon a name that means something to everybody. They call their proposed courses Basic Mathematics 1, 2, 3 etc. I shall give you some mimeographed sheets that will give you the contents of each stage. This may not agree with your ideas or be useable in your school, but at least it is a start.

Besides a name for the course or courses, there are other divisions to the problem that demand attention, namely—text books, personnel of the classes, administrator, counselor, the parents and teacher.

Upon looking over some twenty-five text books whose titles intimate subject matter usable in the non-college preparatory classes one comes to the conclusion that, no one book will be satisfactory for the class, because the content of the course will of necessity need to be varied according to the community, the school, and even according to the personnel in each class. Your own school system may not sanction the use of five sets of books each with different content, to be used in the same class. The cost of this situation may be an item which could be used to show the excessive cost of schools and will therefore be frowned upon by some of our elected school officials. The solution of this part of the problem lies with a good keen minded teacher who will choose the material and assemble it into a loose leaf mathematics book. This is a big job for any teacher and cannot be negotiated without the aid and cooperation of clerical aid. It might be well to enlist the services of the commercial department in this project and both the mathematics and commercial department benefit by it.

The problem as to the personnel of these classes and the selection of the personnel is one that should receive very careful consideration. The average junior high school permits only the best mentally qualified to attempt algebra. This number is only a small percentage of those who go to senior high school. The pupils who were not permitted to take algebra in the junior high school arrive at the senior high school level with their adolescent 'ego' at its peak. This 'ego' has prompted its possessor with the idea that he is now ready to begin his trek toward college. Little thought is given to the requirements of colleges in the way of grades. No stock has been taken of his financial possibility of going to college. He has suddenly blossomed into a mental giant, able to surmount any curriculum difficulty and join the parade of the selected few toward college. The laws of heredity are not so easily changed, neither are work habits and scholastic accomplishments. It is not reasonable to assume that the transition from junior high school to senior high school is the motivation or mental vitamin that will cause the improvement of mentality or work habits sufficient enough to metamorphose the junior high school pupil of questionable ability into a senior high school pupil of college potentialities. It may be said with some degree of certainty that the junior high school pupil's ability for abstract thinking has not as yet developed and therefore a course in non-college preparatory mathematics would be advisable. When the junior high school pupil comes to senior high school and asks admission to First Year Algebra, there are four facts, I would like to know before granting the request or advising against it.

1. Intelligence quotient
2. Reading grade placement
3. Grade placement on some standardized arithmetic test
4. A statement from his junior high high school mathematics teacher as to his work habits and the possibility of his success or failure in academic mathematics.

These four facts plus the judgement of the Mathematics Department of the senior high school, would determine the advisability of the pupil pursuing an academic course. In most instances these could be gotten from the junior high school accumulative record card before the pupil was programmed for academic mathematics. If not available there, during the first

week of the B-10 semester in senior high school, this data could be gotten by giving tests in all Algebra I classes. Then his program could be changed if the need was apparent. It must be definitely understood, that if a pupil demands to take academic mathematics, even though all evidence points to the contrary, he must be given the opportunity because we are teaching in free public schools supported by the taxes of John Q. Public. However, the pupil may be told that if he attempts academic mathematics, he must satisfy the instructor that he can make good by the end of the first five week period or he will be transferred to non-college mathematics for further basic training. This will necessitate two procedures. First, the Algebra I classes must be overloaded at registration and continue so for the five weeks period. The non-college mathematics classes must parallel the Algebra I classes in periods, so that the change from the academic to the non-academic math may be made with the least disturbance to other classes in the program. This plan can be accomplished in the larger high school without an increase in teacher time and with great improvement in the quality of the material in the academic classes. It must be realized that the above plan will not be as flexible in a smaller high school. Many pupils from the junior high school will enter the non-college mathematics classes voluntarily when they are made to understand that their individual needs will be more readily satisfied by these classes. The pupil should be assured that enrolling in a non-college mathematics class does not bar him from taking academic mathematics at some future time. It is a fact that lack of proficiency in the fundamentals of arithmetic is a definite handicap to a pupil entering an academic mathematics class and this lack alone may be the difference between success and failure in academic mathematics The Industrial Arts Major and the Commercial Major will desire to enter the non-college mathematics class in order to get a good review of basic fundamentals and receive instruction in mathematics pertaining directly to their major field.

The administrator must realize the need for the non-college preparatory mathematics and be convinced that there are enough pupils to warrant the creation of class or classes or all effort will be in vain. The administrator has his problems arising from this mathematics. Each department in the school has conceived some panacea for the problems in the school in particular and the world in general. A good many times their panaceas are selfish personal interests of some teacher or a department. He must listen to all the panaceas and select those which will benefit the school as a whole and fit into the overall picture of his school. He is goverened by laws and regulations of a central office which is fortified by statisticians who can prove or disprove anything by means of their statistics. For the most part these statistical wizards have been away from the classroom so long that they cease to realize human beings with diverse abilities gather there for instruction. These human beings have been reduced to abstract norms, medians, deviations, and parts of tax dollars and cease to be individuals with many and varied mental capacities. They become an inanimate pawn in this administrative game of educational chess. Little place is given in the analyses for the actual need of these non-college mathematics derelects. Until we can show in some concrete form the value of the non-college preparatory mathematics to the future of society, we will not be able to combat these multicolored graphs, many times clothed in a false economy robe used as a ruse to get votes and only becloud our problem. Since the colleges are not interested in the non-college preparatory pupil as a prospective student, they have never gathered any data which might predict his value to society after he had been given the opportunity to pursue a curriculum that satisfied his needs. But when the college informs the administrator that pupils from his High School are in the lower quartile of their mathematics classes, he becomes

alarmed and action is started. If our non-college mathematics pupils, as they grow to maturity and fill their niche in society would verbally record their feelings in regard to their neglected plight in mathematics and present them to a meeting of administrators such as are in session in this city now, I suspect that action would start and at once.

The counselor of the school has duty to perform for these non-college preparatory mathematics pupils. Up to now, the majority of the non-college mathematics classes have been a dumping ground. A sump pool into which the counselor could place all the misfits and thus reduce the immediate pressure on his time and energy only to create discipline cases that sap the vim and vigor of teacher and vice-principals. This I protest vehemently. It does not solve the problem it makes it more complicated. My protest has not been long nor loud enough because not much has been done to remedy the situation. With the problem becoming more acute, through your effort and mine, the counselor will be forced to contribute a definite part in eliminating this out of sight out of mind way of handling the situation. The counselor must not only look at the immediate disposition of a pupil but look ahead and see if his placement now will aid in the future betterment of the pupil. Does the non-college mathematics class after due consideration of the pupil's needs seem to be specifically related to enabling him to solve some of his own problems in his future daily life. Let's not say you do not fit into a academic class. Let's say academic classes in mathematics do not seem to meet your present and future needs. Let's build up his confidence. Too often the non-college pupil has been made to feel that he is a square peg in a round hole and has been the cause of weakening the sinews of our educational system. Let's make the pupil feel wanted and we as educational engineers will build him an educational structure that will meet his needs rather than ask him to be content with a structure that does not meet his needs now or in the future. Let's place him in a mathematics class that will give him the opportunity to learn something about interest rates, pay rolls, taxes and many other problems that will aid him in protecting himself against the human hawks and sharks that prey upon his kind.

Last but not least is the teacher of this non-college preparatory mathematics. The last teacher that comes into the mathematics department is the one by virtue of the time of arrival, who gets the non-college mathematics. Too often the college preparatory mathematics teachers have considered the non-college mathematics as a menial task and very subservient to their training in vectors and calculus. The truth is, that the college preparatory mathematics teacher has discovered that it takes the best a teacher can give to cope with the discipline problems and at the same time put over a constructive program. You must be able to do more than assign a lesson and correct answers. You must have something to sell and then sell it. You must be on the job for a full class period, eager to aid each individual pupil. The non-college preparatory mathematics teacher must be one of broad experience, sympathetic, a practical psychologist, and be intensely interested in the subject and be prepared to work long hours gathering and assembling material for this heterogenous group with kaleidoscopic needs. Compensation for this salvage job will be the regular salary of any other teacher according to the salary schedule in your school. The gratitude of returning pupils on Alumni Day will compensate you a thousand fold, and you will not need to enter it on your income tax return. Not because it has no intrinsic value but because no one can estimate the value in exact dollars and cents, of your part in making a good citizen. A girl occupying a position in an office will say that she is very glad to have had the opportunity of studying decimals and percentage in your class because it aided her in being successful in her job. A boy home on leave from the U. S. Navy will tell you that the aid you gave him,

enabled him to pass his test for Diesel School. An Academic math major who took your class as an elective will tell you how his review of decimals was a boon to him in Calculus.

It seems to me with a non-college mathematics program geared to the needs of the individual pupil, will do more to make John Q. Public feel friendly toward teachers, schools, and school taxes than the college preparatory mathematics classes constructed for the elite few.

Won't you please give this non-college preparatory mathematics course more than just lip service in your school? Nominate yourself a committee of one to get some action started. It is a lot of fun, and if you believe me, the satisfaction derived from aiding some individual to understand and eventually like mathematics and change him from a disgruntled potential ne'er do well' to a citizen that is respected in a community, will more than repay you for the effort you have expended. Let's all join in one concerted effort with a united front and start action on this problem. It can be done and with your aid it will be done.

PROPOSED SEQUENCE OF COURSES FOR NON-COLLEGE PREPARATORY MATHEMATICS

Basic Mathematics 1A

Remedial in nature for low I.Q. pupils and those who fall below the required norm in the achievement test given late in A9 or beginning of B10.

Topics

1. Fundamental processes of whole number, decimals and fractions.
2. Simple business practices.
   a. Buying and selling
   b. Budgets
   c. Taxes
3. Simple measurements.
   a. Perimeters
   b. Areas

Basic Mathematics 1B

For the pupils of Basic Mathematics 1A who have failed to attain sufficient mastery of the material in Basic Mathematics 1A to qualify for Basic Mathematics II

Topics

1. Fundamental processes of whole numbers, decimals and fractions.
2. Simple business practices.
   a. Buying and selling
   b. Budgets
   c. Simpe interest, commission and discount
   d. Taxes
3. Simple measurements and formulae.
   a. Perimeters
   b. Areas
   c. Volumes

Basic Mathematics II

To take the place of Applied Mathematics I

Topics

1. Fundamental processes of whole numbers, mixed numbers, decimals and fractions.
2. Business arithmetic and percentages.
3. Measurements of lines, areas and volumes.

Basic Mathematics III

To take the place of Applied Mathematics II

Topics

1. Review of fundamental processes.
2. Decimal, fraction and percent conversion.
3. Mensuration, tables and conversion.
   a. Formula
   b. Areas, and volumes
4. Squares and square root.
   a. Law of Pythagoras
5. Graphs, tabular values and scale drawings.

Basic Mathematics IV

To take the place of Senior Mathematics.

Topics

1. Fundamental practices in whole numbers, decimals and fractions.
2. Business practices.
   a. Percentages
   b. Buying and selling
   c. Inventories

d. Payrolls
e. Profit and loss with overhead cost
f. Bank accounts
g. Discount and commission
h. Insurance
i. Stocks and bonds
j. Time payments
k. Taxes

3. Simple statistics, graphs and tables.
   a. The making and interpreting of tables

BASIC MATHEMATICS V

For pupils who have satisfied the requirements of previous courses in Basic Mathematics and who are not going to college.

Topics

1. Formulas, evaluation and equations.
2. The four, fundamental processes in Algebra with signed numbers.
3. Use and handling of parentheses.
4. Tables of weights and measures, English and metric with conversions, English to metric and metric to English.
5. Use of ruler, compasses and protractor in scale drawings.
6. Simple geometric construction.
7. Plane figures and simple solid figures.
8. Formulas of the square, the equilateral triangle, the 60°–30° right triangle and the 45° right triangle.
9. Right triangle including trigonometric solutions.
10. Vectors and the parallelogram of forces.
11. Practical applications of mensuration.
12. Optional-Logarithms and interpolation.

Ideal Classroom*

By RICHARD MORRIS
Rutgers University, New Brunswick, N. J.

THE chief problem of education is not that of imparting knowledge; neither is it essentially the effort on the part of the student to acquire learning himself, although both of these activities are highly important, each supplementing the other, whether the legacy is from the teacher or the printed page. The possession of knowledge gives the possessor a leverage that should mean power if rightly and wisely employed. Similarly, the ability to acquire knowledge measures the mental acumen of the operator. But education, in the broadest sense, transcends both of these phases. The educational structure cannot be perfectly erected without these, and yet in so many instances it is far from perfection for the want of something additional. Environment plays a part, so does opportunity, so does vision, and driving motive.

The great leaders of humans have not always been those most highly educated in the academic sense, neither have they always been the most perfect, often were they decadent leaders. But history will bear record that the ablest and most idealistic leaders have been those persons most highly educated in the broadest sense. All of this reasoning lends emphasis to the fact that the classroom is the laboratory where the individual may discover whether he possesses qualities for leadership. Leadership means pointing the way for others. The well educated person, the person with a broad vision of human relations and affairs, the one who has all of his powers well in hand, the one who can assemble all of the relevant facts about a problem or situation and can see a way out, is very apt to be a good leader.

The ideal classroom is the place where this kind of training is taking place every day. I think that all of us as teachers, are endeavoring to make the classroom a means to an end, viz., aiding pupils to discover themselves, helping them to acquire knowledge and skills, urging them to develop latent abilities, and pointing out indirectly that the classroom presents an opportunity and not just a burdensome task. Training for leadership is not necessasily a conscious effort per se. The leader has been endowed from the beginning, and it rests with him and his environment to determine whether his talents shall be put to use or be buried in a napkin.

* Reprinted from *Rutgers Alumni Monthly*.

The Unit Column vs. the Decimal Point

THE ROLE OF PLACE IN WRITTEN NUMBER

By HIRAM B. LOOMIS
Chicago, Ill.

THE importance of place in written number is evident when we realize that it is our Arabic system of writing number with zero to indicate the empty place or column that enables pupils in our elementary schools to perform arithmetical operations that would trouble good mathematicians confined to the Roman system. In addition and subtraction we have practically been forced to realize the desirability of keeping units digits under units digits and tens digits under tens digits; and in bookkeeping we have paper ruled for this purpose. The objective of this article is to present the desirability of training pupils to use similar columns for all written work in arithmetic; to put every units digit in the units column, every tens digit in the tens column, every tenths digit in the tenths column etc.: and to do this in multiplication and division as well as in addition and subtraction. This will eliminate the problem of "pointing off," for the decimal point will necessarily be between the units column and the tenths column for every number on the paper.

If this is taught from the beginning, and if the mechanics are no more difficult than at present, it will prove a decided gain, for "pointing off" is a stumbling block. But more fundamentally it will eliminate an idea of place introduced by the decimal point that is foreign to the spirit of arithmetic, and substitute for it the idea of place upon which our Arabic system of writing number is founded, a place for units, a place for tens, a place for hundreds etc. The Arabic system itself is based on our method of counting, for we count by units, by tens, by hundreds etc. Pupils will come to think in terms of column, to think of number in the way in which we count, and in the way in which we put the count on paper. They will see in columns exactly what the user of the abacus sees in the wire on which the beads are strung.

In support of my contention that the decimal point introduces an idea of place foreign to arithmetic, I shall at first simply give two illustrations, later I shall at least indicate a philosophical reason.

The decimal point makes an asymmetric, and the units column a symmetric, division of written number. This appears clearly in the illustration below:

Thousands hundreds tens units
↑ ↑
tenths hundredths thousandths

Going to the left from the units column we increase by powers of ten; going to the right we decrease by powers of ten. We have something like a geometrical progression; but the decimal point is outside of the progression. It is in the "Inbetween," in the land of "Nowhere." Before we can do business, before we can either multiply or divide by ten, we must get inside of the progression, we must get inside the domain of arithmetic.

The second illustration is a comparison of two rules for getting the characteristic of a logarithm. One rule counts from the decimal point; the other, from the units column. The following rule is from a standard text on trigonometry:

When the number is greater than 1, *the characteristic is positive, and is one less than the number of figures to the left of the decimal point; when the number is less than* 1, *the characteristic is negative, and is one more than the number of zeros between the decimal point and the first significant figure.*

As I have never seen in print a rule that counts from the units column, I submit the following home-made rule:

If the first significant figure is in the units column, the characteristic is 0; *if it is A*

columns to the left of the units column, the characteristic is plus A; if it is A columns to the right of the units column, the characteristic is minus A.

Notice how the second rule dovetails with the whole system without any adjustment, without any adding or subtracting of one. 10 is the first power of 10; and if the first significant figure is in the tens column, the characteristic is plus 1. 100 is the second power of 10; and if the first significant figure is in the hundreds column, the characteristic is plus 2. 1/10 is the minus first power of 10; and if the first significant figure is in the tenths column, the characteristic is minus 1. This complete dovetailing compared with the adjustments needed in the other rule at least deserves consideration. How many mistakes have been made from not knowing whether to add or subtract one?

When I begin this paper I had no idea of its being more than a mere device for handling decimals; but the further I got into it, the more it seemed to deal with fundamentals. The idea first came to me by accident. I was preparing a discussion of the proposition of Marx Holt, Principal of the Fisk School, Chicago, to substitute for our decimal system a number system on the base eight, itself resting on a number system on the base two. The binary system is preeminently a place system. Every place (column) is either occupied by the digit, 1, or is empty. Of it no less a mathematician than Leibnitz has written, "Calculation on the base two, that is by the use of the digits 0 and 1 only, compensates for its length by being the most fundamental for science and yields new discoveries that have useful applications."

In my discussion I wanted side by side the same examples in multiplication and division on the three number bases, 10, 8, and 2. I had to crowd things. I did so by using columns and putting every digit in its proper column. I happened to notice how the decimal point had taken care of itself. It was always between the units column and the tenths column. My first thought was that this method of putting work on paper might be a good device for avoiding difficulties in "pointing off." Later I came to the conclusion that it would also tend to a better understanding of the fundamentals of arithmetic.

In what follows I shall first give a possible method of presenting multiplication and long division by using paper ruled in columns and by putting every digit in its proper column. I shall then add some remarks on the nature of arithmetic.

Multiplication on Paper Ruled in Columns

		3 2		
	3	6 2		
3	6	2		
6	3 2			
6	3			2
			6	

The units column is doubly ruled.

1. Multiply 3 units by 2 in the units column. The product is 6 units.

2. Multiply 3 tens by 2 in the units column. The product is 6 tens. I am not putting zeros in the empty columns. I do this to emphasize PLACE.

3. Multiply 3 hundreds by 2 in the units column. The product is 6 hundreds.

Up to this point the multiplier has been in the units column. We shall now try it in other columns.

4. Multiply 3 tens by 2 in the tens column. The product is 6 hundreds. Two facts enter into the operation. The multiplier is 2 and the 2 is in the tens column. We first multiply by 2. 2 times 3 tens is 6 tens; and 6 tens times 10 (the tens column) is 6 hundreds.

5. Multiply 3 tens by 2 in the hundredths column. The product is 6 tenths

We get it in two steps. First we multiply 3 tens by 2 and get 6 tens. Then we multiply 6 tens by 1/100 (the hundredths column) and get 6 tenths.

We are now in position to make a rule of thumb. To multiply a digit in the multiplicand by a digit in the multiplier, first multiply the two digits; then if the digit in the multiplier is in the units column, put the product in the same column as the digit in the multiplicand; but if the digit in the multiplier is A columns left (right) of the units column, put the product A columns left (right) of the digit in the multiplicand.

Let us next take the general case. Multiply 60.34 by 72.08. In figure 1 the product of each pair of digits is entered separately. In figure 2 they are united to get a form essentially the same as that of our present practice.

Fig. 1

		6	0	3	4		
		7	2	0	8		
						3	2
					2	4	
			4	8			
					8		
				6			
	1	2					
			2	8			
		2	1				
4	2						
4	3	4	9	3	0	7	2

Fig. 2

		6	0	3	4		
		7	2	0	8		
			4	8	2	7	2
	1	2	0	6	8		
4	2	2	3	8			
4	3	4	9	3	0	7	2

Notice that the decimal point takes care of itself. It is between the units column and the tenths column for every number on the paper. If this form were taught from the beginning, I believe it would become just as habitual as the present form of writing partial products on the bias is today. At all events, if it became habitual, the decimal point would lose some of its terrors.

Long Division on Paper Ruled in Columns

As division is essentially the undoing of multiplication, I shall use as my example the undoing of the multiplication just performed. I shall also use the form of multiplication to the extent of putting the dividend where we had the product, the divisor where we had the multiplicand, and the quotient where we had the multiplier. Our problem is to divide 4349.3072 by 60.34. We set it down as in figure 3.

Fig. 3

		6	0	3	4		
4	3	4	9	3	0	7	2

Fig. 4

		6	0	3	4		
		7					
4	3	4	9	3	0	7	2
4	2	2	3	8			
	1	2	5	5	0	7	2

Fig. 5

		6	0	3	4		
		7	2				
4	3	4	9	3	0	7	2
4	2	2	3	8			
	1	2	5	5	0	7	2
	1	2	0	6	8		
			4	8	2	7	2

Fig. 6

		6	0	3	4		
		7	2		8		
4	3	4	9	3	0	7	2
4	2	2	3	8			
	1	2	5	5	0	7	2
	1	2	0	6	8		
			4	8	2	7	2
			4	8	2	7	2

We begin just as at present by asking how many times 6 is contained in 43. The answer is 7. 7 times 6 is 42. The 2 of the 42 will naturally come under the 3 of the 43, which is one column to the left of the 6 in the divisor. Therefore, according to our rule of thumb for multiplication, the 7 of the quotient must be one column to the left of the units column. We put it there,

multiply the divisor by it, and subtract, as shown in figure 4.

We next ask how many times 6 is contained in 12. The answer is 2. 2 times 6 is 12. The 2 of the 12 comes under the 6 of the divisor, therefore the 2 of the quotient must be in the units column. We put it there, multiply the divisor by it, and subtract, as shown in figure 5.

We next ask how many times 6 is contained in 48. The answer is 8. 8 times 6 is 48. The 8 of the 48 comes two columns to the right of the 6 in the divisor, therefore the 8 of the quotient must be two columns to the right of the units column. We put it there, multiply the divisor by it, and subtract with no remainder, as shown in figure 6.

To emphasize the emptiness of the tenths column of the quotient, it has been left empty. It may be filled with a zero.

This form differs so slightly from the ordinary form of division that further details seem unnecessary. It should be noted, however, that every number in the operation may be properly "pointed off" by putting a decimal point between the units column and the tenths column.

The Nature of Arithmetic

The fundamental reason for advocating the use of columns is the fact that thinking in terms of columns is in harmony with the nature of arithmetic. The last chapter of Dantzig's *Number the Language of Science* has for its title "The Two Realities." Our inability to stand at both poles of reality at the same time has shown itself from the time men began to think, and crops that everywhere to-day. Our educators talk of a "changing" as opposed to a "stable" world; of a "dynamic" as opposed to a "static" universe. In mathematics the problem takes this form: Do we live in an atomic universe or in a universe that is a continuum? The world of arithmetic is atomic; the world of geometry, a continuum. Arithmetic is based on counting, and in counting we jump from one to two, and from two to three. But in Geometry we glide along a line or over a surface without any breaks.

Perhaps the quandry in which modern physicists find themselves may help us to realize our own difficulty. A few years ago physicists were practically agreed that the wave theory of light had supplanted the old corpuscular theory. Now, however, since Planck's discovery of the quantum, the situation is such that Eddington writes in his Nature of the Physical World, "Sir William Bragg was not overstating the case when he said that we use the classical theory on Mondays, Wednesdays and Fridays; and the quantum theory on Tuesdays, Thursdays and Saturdays." To illustrate, a radio message leaves the sending station in whole quanta, never a fraction of a quantum (i.e., in atoms). It goes out into space as wave motion. If it keeps going, its intensity may be reduced to any degree, not limited by the size of the quantum. But before it can be received at the receiving station, whole quanta must be gathered together. Part of the week our physicists live in an atomic universe; the rest of the week, in a continuum.

I believe that in using on Monday the theory they know will work on Monday, and on Tuesday the theory they know will work on Tuesday, our physicists are wise. Since arithmetic is essentially atomic, I advocate living in an atomic universe while working arithmetic. Arithmetic is a calculus of integers. As I use the term, arithmetic is the branch of mathematics by which we derive other numerical values from data given in business, in science and in engineering.

But teachers, business men, scientists and engineers insist that they must have fractions. They ask me to be practical. I accept the challenge. However, I shall insist on being practical from the ground up. I shall take fractions from them. I shall ask them how they got their fractions. Then I shall treat fractions in exactly the way in which they got them.

I give the business man some currency

and change, and ask him to count it. He reports, "Four dollars and sixty-nine cents." I note first that he reports in two units, and gives an integral number for each unit. However, we will write the 69 cents,69/100 of a dollar. I then want to know where he got the 100, what he did with it, and how he got the 69. I find that he simply took the 100; that he imagined the dollar divided into 100 EQUAL parts; and then that he counted tokens that represented these equal parts. He made a smaller unit, the cent, and counted the number involved. He got his fraction in the only way in which a fraction can be got in arithmetic. He denominated the number of equal parts into which he imagined the dollar divided, and then he numerated such of those smaller units as were involved.

We reach the same conclusion in dealing with the scientist or engineer. I draw a line on paper, give one of them a scale, and ask him to measure it. He says, "Three and 7/16 inches." The scale was divided into inches, and counted three inches with a remainder. The fourth inch of the scale was divided into 16 equal parts, and counted 7 of these smaller divisions. The scale denominated 16 and numerated 7.

Let us next take a really critical case, the ratio of the diagonal to the side of a square. This case is of historical importance, because the discovery that this ratio is incommensurable threw consternation into the Pythagoreans of ancient Greece. The ratio, $\sqrt{2}$, is easily expanded into the intermediate decimal, 1,414213 · · · · No one can work with such a decimal. What we do, is to cut off all but a few digits after the decimal point and throw them away. In effect arithmetic says, "I can deal only in integers (atoms). I will divide the side of the square into 10 equal parts, into 100 equal parts, into any number of equal parts you want. I will then count the number of these smaller units contained in the diagonal and throw the excess away." Arithmetic will give to science or business calculations that match in accuracy any data they can give. Arithmetic cannot get outside the domain of integers in making calculations; neither can science or business get outside the domain of integers in making their smaller units or in counting them.

In short, a fraction is simply an integral number of a unit smaller than the principal unit. When fractions are used, at least two units are likely to be involved. For example, 7 lbs. 12 oz., $7\frac{3}{4}$ lbs., and 7.75 lbs. denote the same quantity. The first is evidently expressed in two units. The second means 7 lbs. and 3 "quarters of a pound", the third, 7 lbs. and 75 "hundredths of a pound," In each case two units are involved. Passing from concrete to abstract number, we meet this same multiplicity of units. The number 4,563 is read 4 thousand—5 hundred—6ty (tens) – 3 (ones). I used the word, ones, because I want to save the word, unit, to designate the thing we count, miles, pounds, dollars, apples, tenths, tens, etc. The column in our written number is essentially a denominator. It names the unit that is being counted. If a fraction is added to an integer, we simply add a column to give the size and number of the smaller unit involved.

So ingrained is this idea of units (with their columns) in the very nature of number, that we group columns into larger columns just as we group smaller units into larger units. Take the number 123,456.789. Note the division by the comma as well as by the decimal point. Note how we read it. One hundred twenty-three thousand—four hundred fifty-six—and seven hundred eighty-nine thousandths. Place (columns) is of the very essence of our written number.

Not only have we failed to take full advantage of place in our teaching and practice of arithmetic; we have even introduced a foreign idea of place, the decimal point. For the decimal point is outside the domain of arithmetic, outside the world of atoms. It is in the "In between." Nor is it really at home in the continuum, for zero, which separates positive from negative

number, makes the natural symmetric division there. Its excuse for existence (a perfectly natural excuse) is that it is easier to make one dot than two. It would take a dot on each side of the units column to locate the natural center. The best thing we can do with the decimal point is to use it to locate the units column, and then forget it. If we need it in our result, we can put it between the units column and the tenths column. But our thinking and our work should be done in terms of columns, in terms of place that fits the nature of arithmetic.

I should not feel that I had even approached justice to the importance of place in written number, did I not venture a prophecy, for I believe that an octic system based on the binary system will eventually take the place of our decimal system, and that it will prove as great an advance as was the introduction of the Arabic system. If the seven digits used to write the octic system are, as Mr. Holt suggests modified forms of the first seven numbers written on the binary system, our numbers will be written on both the octic and the binary system at the same time. When we can begin teaching on the binary system, a pure place system with but one number fact (one plus one equals two), and build our octic system on that, the simplification of arithmetic will result in mathematics playing a much larger part in all lines of work. Moreover we shall not only have all the advantages that mathematicians have seen in an octic system, but also the advantages that Leibnitz saw when he wrote the passage referred to earlier: "Mais le calcul par deux, c'est-a-dire par 0 et par 1, en recompense de sa longueur, est le plus fondamental pour la science, et donne de nouvelles decouvertes, qui se trouvent utiles ensuites, meme pour la pratique des nombres, et sur-tout pour la Geometrie; dont la raison est, que les nombres etant redruits aux plus simples principes, comme 0 et 1, il paroit partout un ordre merveilleux."*

* Memoires de l'Academie Royal des Science, 1703, page 87.

Geometry

By Katharine O'Brien, Portland, Me.

We agree not to define
Point, and plane, and single line.

Other terms and requisitions
We pin down with definitions.

We assume some postulates
As demand necessitates.

Given an hypothesis,
(Someone else can hand us this)

We proceed without confusion
To a logical conclusion.

Thus we prove a proposition—
This the goal of our ambition.

I Had the Craziest Dream

By
7B's at Taft Junior High School, *Washington, D. C.*
and
Mrs. Faith F. Novinger, *Teacher of Mathematics*

Announcer: (Off stage) The name of our play is . . . .

The pianist plays "The Craziest Dream," by Harry Warren.

Announcer: Yes, that's it. That is the name of our play. The characters are:

Our Hero: John Jones (walks across stage in front of curtain to this is "The Craziest Dream").

Pupils: Bob, John, Sue, Katherine, and George.

Teacher

Navy Recruiting Officer (Carries placard.)

Parachute Riggers

Aircraft Armorer

Mapmaker

Weatherman

Grocer

Banker

Scene I

Loud Speaker: Ah, the scene is a school room. A Math class is about to begin. See if you can find our hero John Jones. (John and Bob walk on to the stage.) Bob is carrying some text books.

John: Slave, carrying all those books.

Bob: Didn't you do your homework?

John: Heck no, I'll have plenty of time after I get to class. (John takes the last seat in the row and barricades himself behind funny books.)

Teacher: John, you may start reading the answers to your homework.

John: (stands, dropping funny books) I haven't done it yet.

Teacher: Why didn't you do it, John?

John: Oh heck, I hate Math! What good is Math to you when you grow up?

Teacher: John, don't you know that Math is used in almost every kind of work there is? It's used by weather bureau forecasters, architects, gunners, mapmakers, airplane pilots, navigators, stenographers, factory workers, storekeepers, bookkeepers, and many others. Today we will review, naming and measuring angles. Will you name this angle, Sue?

Sue: It is a right angle; it has 90°.

Teacher: (holds up obtuse angle) John.

John: Scalene.

Teacher: No, Katherine.

Katherine: Obtuse. It has more than 90° in it.

Teacher: (points to triangle on the board) Bob, you will measure the angle at A.

Bob: I place the arrow of the protractor at the vertex of the angle A. It is 90°.

Teacher: Write down angle A equal to 90°. George, Measure angle B.

George: It is 30°. Write angle B equal to 30°.

Teacher: Mary, measure angle C.

Mary: Angle C equals 60°.

Teacher: Susan, find the sum of the three angles.

Susan: The sum of the angles in this triangle equals 180°.

Teacher: Draw any triangle for yourself. Measure the angles and find the sum. See if it is equal to 180°, too. I want you to pass this triangle and rectangle around, and find the difference between them.

Bob: The triangle is rigid, the rectangle collapses. But we could brace it.

Teacher: Then what would you have?

Bob: Two rigid triangles. (In the meantime John has gone to sleep.) (Bell rings.)

Teacher: Class excused. (Class leaves. John sleeps on.)

Dim lights.

Loud Speaker: John is asleep. Here is what he is dreaming.

John: Mrs. Simmons thinks I am no good, that I don't know my mathematics. I wonder if that's true, what that teacher

said, or is she saying that just to make me do my Math. I wonder. I am 15. I'll quit school, get a job, and make 40 a week. I'll show her up. Math! Who needs to know the stuffy old stuff anyway? Bah!

The following enter stage from right and form a line: *Navy Recruiting Officer, Parachute Rigger, Aircraft Armorer, Mapmaker, Weatherman, Grocer, and Banker.*

Navy Recruiting Officer steps forward. (Spot each interview between John and Officer, etc.)

John: Mr. Navy Recruiting Officer, I am a fine young man, healthy, and I want a job piloting airplanes. When do I begin?

Navy Recruiting Officer: Do you know your Math? Can you read a protractor accurately? Do you realize that a 2° mistake on a 700 mile journey means that an airplane will be 24.5 miles from its ship and you cannot even see your ship that far away in hazy weather? A misplaced decimal point might leave the plane out of the maneuvers entirely. No, you will not do for us. You cannot do your protractor work.

John: Mr. Parachute Rigger, I'd like to rig up parachutes. I always did like to see those hemispheres floating earthward like toadstools.

Mr. Parachute Rigger: Men's lives depend on these parachutes. We have to pick out men well trained in the fundamentals of mathematics. To be a good parachute rigger you have to know Algebra, be able, to solve equations and formulas. You have to know the figures of Geometry. You have to know circular measure. Do you know these things?

John: I haven't had Algebra yet, I'm only in the seventh grade.

Parachute Rigger: You don't qualify for a parachute rigger; you don't know your Math.

John: Mr. Aircraft Armorer, I'd like to inspect, adjust, and repair armament equipment, machine guns, cameras and bomb racks. Will you give me a job?

Armorer: You look all right, but do you know it's not so easy to inspect the armor of a plane? You must know trigonometry, mechanical drawing, and blue print reading. Did you learn these things in high school?

John: You make it sound hard. I was never one to waste my time on Math.

Armorer: Too bad, sir, we cannot waste our time with you. You don't know your Math.

John: Mr. Machinist, I want to be the man behind the man behind the guns. I want to be a machinist.

Machinist: Do you know your Math? Do you always put the decimal point in the right place? Can you measure with precision to the thousandths of an inch? One hair is .003 of an inch, and some parts won't fit if they are not accurate to one thousandth of an inch. Can you use a slide rule? Can you use a micrometer?

John: Ouch! not decimals. They're terrible.

Machinist: I can't use you in this war.

John: Mr. Mapmaker, may I work for you?

Mapmaker: Can you draw?

John: I can draw funnies. (*Aside.* I've read them enough.) Can't I draw maps for the Navy pilots?

Mapmaker: Were you careful and accurate in your scale drawings at school?

John: I was sort of good.

Mapmaker: Sort of isn't good enough, for our army and navy. They are the best in the world, and we must give them perfect maps. I can't use you.

John: Please, Mr. Weatherman, I'd like to learn to use your funny instruments. I want to be the man who tells the airplane pilots the weather conditions.

Weatherman: Do you know your solids, similar figures, and angles? Can you measure degrees?

John: Not yet, sir. I was supposed to be learning that, when I started this dream.

Weatherman: Come back when you have learned it.

John: Mr. Grocer, my name is John Jones and I'd like to have a job.

Grocer: How is your Math?

John: Not so good. You see I never liked it much.

Grocer: You never liked it much? Do you know how Math is used in the grocery business?

John: Why, no, sir.

Grocer: You must add the sales slips accurately. You must be able to make change correctly. You must be able to check invoices and figure per cent of profit. No, I fear you cannot have this job.

John: Mr. Banker, let me work for you.

Banker: Can you do Math? Did you take bookkeeping? Can you add, subtract, and do percentage?

John: No.

Banker: I cannot use you.

Cut off spot.

Choral speaking: You cannot add accurately. You cannot subtract exactly. You cannot multiply, divide or measure. We cannot use the man who doesn't know his Math.

(Repeat faster tempo and higher 3 times. Shake hands at John and close in on him.)

John: (hysterically) Mrs. Simmons, Mrs. Simmons, Oh, Mrs. Simmons! I want you to teach me Math! Please teach me Math. You just gotta teach me Math!

Lights. Enter teacher.

Teacher: Why, John, I'll gladly teach you Math. I'll teach you to add, subtract, multiply, and divide. I'll teach you to measure lines by inches, feet, yards, miles, by centimeters and meters. I'll teach you to measure by pounds and ounces, by degrees and minutes. But John, I can only teach. You, and you alone, can learn as much as you are willing to study hard to get.

John: I'll do it. I'll learn to add, multiply, divide, and subtract. I'll learn to measure in feet, yards, ounces, pounds, degrees and minutes.

Teacher: And then John there is High School, Algebra, Geometry, and Trigonometry.

John: I'll do it.

Loud speaker: John learns his tables. He learns to add accurately. He can measure by yards, by degrees, and minutes. He is super. In fact he is a super mathematician.

Curtain.

SCENE II

Loud Speaker: Time later. This is an airplane factory. Mr. Old Hand is talking to Mr. New Hand.

Old Hand: We are building this bomber for Ace John Jones.

New Hand: Yeow! John Jones? Did you know he went to the same school I did? He was a couple of years ahead of me. We called him super, because he was a super mathematician. He really did study hard.

Old Hand: Super, super mathematician, super ace, nine planes last week. This plane will be ready for him next week. Here is the model of the plane. Notice how rigid the planes are made by using triangles in steel construction work.

New Hand: That sounds like Math to me. My Math teacher taught me about rigid triangles in the seventh grade.

Old Hand: Then to make it even more rigid, we weld the struts to the beams with three holes, because that forms a triangle, too.

New Hand: Let's get the job done.

Old Hand: Here is the blue print. The scale is 1 inch to a foot. If you are a good mechanic on this part of the work you will be promoted to working on the engines.

New Hand: I would like that. I'll do my best to help win this war. (Show riveting.)

New Hand: It is nice to know that putting 3 rivets in a triangle makes the planes stronger.

Curtain.

SCENE III

Loud Speaker: We are now at a secret airplane field. Here comes our hero. The bomber is already well used. (John gets in plane, attired in coat and goggles. The navigator and gunner get in too.)

John: Today we are flying Jimmie Doolittle to a secret destination. On the return trip we will drop two 1000 pound

busters on ammunition plants. Gunner, have you checked and double checked your ammunition?

Gunner: Right, Capt. Jones.

John: Navigator, we will be flying above the clouds. I'll depend on you to get us there.

Navigator: Aye, aye, Sir.

Loud Speaker: We are off! It is now 15 minutes later.

John: Come in, navigator.

Navigator: We are now at 60° latitude, 40° longitude, heading 80°, air speed 200 knots an hour. The wind is 30 knots an hour from 340°. My scale drawing shows the ground speed is 208 knots and our true course is 88°.

John: We are over enemy territory.

Navigator: Altitude 10,000 feet. All is well.

John: We have passed the enemy. General Doolittle will land at latitude 35°, longitude 100°.

Loud Speaker: Our Ace is on the way home to the airport. His bombs have been dropped. The bomber goes in for a perfect landing.

A Conflict Between Theoretical and Computational Mathematics

By Mrs. Alice E. A. Priestley
Wilson College, Chambersburg, Pa.

The distinction between mathematics as a logical, symbolic expression of functional relationships and as a device for computational purposes is frequently misunderstood.

Most people, even some mathematicians fail to see a distinction between the two skills. A theoretical mathematician is frequently expected to make out income tax returns or make change or add a column of figures or to do other accounting tasks. Nothing is more distasteful to a theoretical mathematician. On the other hand, a computer enjoys finding new devices or short cuts or arrangements of numbers and variables.

At present the two interests are in conflict. They should not be. Each has a contribution to make to the advancement of the science. In the hands of the computer the theory becomes useful and applied in a form to defend and promote the general welfare of society.

The conflict is due partly to the misunderstanding of the nature of the two skills of mathematics, and should be resolved in the minds of college students. The conflict could be ended much earlier in the elementary school if more teachers were aware of its nature. Nevertheless something must be done to eliminate the source of friction between mathematics and science departments. One or the other department must decide to make the transfer from theoretical mathematics to computational mathematics for the average student. The brilliant student can make the transfer for himself and does.

The mathematics teacher finds it difficult to make the transfer because his experience in computational mathematics is usually limited to occasional formal manipulation of terms and principles unhampered by the demands of speed and quantity or procedures. Since the scientist has usually had both the academic and practical experience of mathematics, it seems that the transfer should be accomplished by that department.

Computational mathematics has many short-cuts and forms of equations which are very useful and valid for computation. Formulas solved for the unknown variable are the mathematical tools of physics and chemistry. Solving a formula explicitly for each unknown multiplies the forms of the equation for the computer whereas the mathematician is satisfied with one expression of the relationship between the variables. For instance, "distance equals rate times time" expresses a general relationship between the variables, distance, rate and time, and is written symbolically as $d=rt$. For the theoretical mathematician $d=rt$ is the solution of his problem and he has no further interest in it. But the computer must have in addition to $d=rt$ the formulae, $r=d/t$ and $t=d/r$ for use, according to data which have been given him in a specific problem, and according to the computing aids he may be using.

The theoretical mathematician is interested in the most elegant and general form of expressing a relationship between variables. The criteria for elegance and purity of mathematical form is the most general, simple, concise, symbolic expression of a functional relationship between variables and constants derived from a language statement. The computer uses specific expressions modified by given conditions. The specific expression because of the special conditions cannot be manipulated as general expressions free of conditions can be. This misconception of the distinction between general and specific symbolic form creates confusion in the minds of the

average student and makes him feel that theoretical mathematics is a very difficult tool to use. The limitations of manipulation of a specific expression is rarely understood by the student. The mathematics teacher should make clear that rules of computation are dependent upon conditions and the axioms of operation.

The mathematician states a relationship in a concise formula of dependent and independent variables. The computer wants that form of the equation which will facilitate computation of the unknown for him. The properties of the parameters, computing aids or machines used by the computer govern to some extent his choice of form of the equation, thereby making his form a specific one. From a general equation or set of equations many specific forms may be derived by the combining laws of mathematics. The language expression of the specific forms is different and less general than that of the fundamental functional relationship because operational procedures must be included in the former which are postulated in the general expression by the combining laws of calculation and so logically are implied in the theoretical expression.

For instance, the verbal expression of Boyle's law is "the volume of a gas at constant temperature varies inversely as the pressure". In mathematical symbols this is expressed as $V=k/p$. A problem asks for the new volume at a given pressure when the old volume at a known pressure is given. That is, for the mathematician, if $V=k/p$; then $V'=k/p'$; but for the scientist it is $V'=Vp'/p$ or $V'/V=p'/p$. The mathematician solves these two equations by a method of simultaneous equations or by substitution whereas, although the scientist must follow the same mathematical principle, he is apt to state the Law as "the ratio of the volumes equals the ratio of the pressures", thus combining the operational technique with the expression of a simple function.

The connection between these two viewpoints is rarely made by either department and the average students have little imagination or skill in manipulating equations to discover different forms expressing the same relationship or to separate the functional expression from the combining operations.

When the law becomes more involved such as, "Volume varies directly as temperature and inversely as pressure" and the problem is to find V' at a given temperature and pressure when V is known for a known temperature and pressure, the mathematician again writes, if $V=kt/p$; then $V'=kt'/p'$, and solves by substitution. The computer often knows this formula as "new volume is the known volume multiplied by a temperature factor and a pressure factor." Here the student becomes confused and can not remember whether p' is in the numerator or the denominator or which given pressure is the p', and after that he is equally uncertain about t and t'. He can invariably recite the language statement of proportions and frequently can write it in mathematical symbols but finds difficulty in manipulating letters by the combining operations or axioms of algebra. The substitution of numbers for known letters simplifies the expression to such an extent that the average student can solve for the remaining unknown.

Manipulating letters seems to be an abstraction beyond the native ability of the average student but solving numerical equations for one or two unknowns is well within his understanding.

The principle of substitution of equivalent expressions seems to be difficult for the poorer student to understand and accounts for many difficulties. Add Dalton's law of partial pressures to the preceding problem and the difficulty is unreasonably increased. Still, it usually can be quickly translated from "the total pressure exerted by a mixture of gases is equal to the sum of the pressures which each exerts." or symbolically $P_t=P_g+P_{H_2O}$. Solving, $P_g=P_t-P_{H_2O}$ is easy, but to substitute the identity $P_t-P_{H_2O}$ in the preceding for-

mula where P_g is p seems to create innumerable difficulties whereas a numerical value for p does not. The variety of errors which may be made in solving $V=kt/P_t-P_{H_2O}$ and $V'=kt'/p'$ are numerous. This seems to be caused by the difficulty of the mathematical concept of an algebraic sum being manipulated as a simple term rather than as a complex term. The fundamental concepts of identity and operations of addition, division, and fractions do not seem to be real to many students.

To consider a problem in trigonometry it is more practical to subtract 90°, 180°, or 270° from an angle greater than an acute angle expressed in degrees, minutes, and seconds than to risk the chance of error and awkwardness in using denominate numbers. One must make the adjustment for the axis along which the angle is measured according to the quadrant in which the given angle lies. Most mathematical text-books show the angle subtracted from 90° or a multiple of 90°. If the mathematician would write $(n\ 90°+A_2)=A_1$ where A_1 is the known angle, then he could say; when n is even, the function of A_2 is the same name function as A_1 with the sign of the quadrant of A_1; and when n is odd it is a co-function. That is; $\sin 144°\ 27'\ 13''=\sin(90°+54°\ 27'\ 13'')=\cos 54°\ 27'\ 13''$ rather than $\sin(180°-144°\ 27'\ 13'')=\sin 35°\ 32'\ 47''$.

The variety of arrangements of symbols to express any given verbal statement is limited only by the number of variables and constants and operations performed upon them. The computer chooses that formula found to be most convenient for purpose of calculation by whatever mechanical device he uses. The mathematician chooses the most concise, elegant, simple formula expressing the essential functional relationship between dependent and independent variables under the most general conditions. The operational techniques to be used follow by logical implication from special conditions of a problem and axioms of calculation.

The distinction between theoretical mathematicians and computational mathematicians lies in the difference between an interest in the imaginative symbolic expression of an idea and an interest in technique of manipulating such expression by the use of definitions and axioms of operation. The conflict arises because one of these interests is usually dominant in a mathematician. When the first is stronger he is frequently misunderstood until such time as he has acquired prestige as a creative mathematician. It is by his imaginative understanding of the philosophy of mathematics that the great advances are made.

Announcement

BUSINESS EDUCATION VISUAL AIDS, 330 West 72 Street, N.Y. 23, N.Y. has been formed by Clifford Ettinger to rent, sell and produce sound and silent motion pictures, filmstrips, and other visual aids. BEVA will specialize in providing visual helps to high school and college teachers of business, sales managers and training directors. It will restrict its activities to the field of education for business. Selected films and filmstrips from various sources are now available for rental or sale in the fields of accounting, consumer education, sales training, distributive education, duplicating machines, guidance, introduction to business, office machines, shorthand, typewriting, and voicewriting.

Dr. Ettinger has been a business teacher for many years. His thesis, "Projected Visual Aids in Business Education," was completed under the direction of Professors Lomax, Knowlton, and Thrasher. Since 1939 he has been visual education editor of the *Journal of Business Education* and has published over eighty articles dealing with visual aids. During the war he was a project supervisor, education officer, and writer in the Navy Training Film Branch and ended his service with the rank of lieutenant commander.

Let's Explore It With Straight-Edge and Scale

By W. W. Ingraham
Williamstown School, Williamstown, West Virginia

What boy or girl in your mathematics class was not impressed when he read or perhaps just looked at the magazine Life for January 14, 1946, which devoted several pages to a pictorial description of M.I.T.'s "Differential Analyzer." The title catches the eye. "The Great Electro-Mechanical Brain," "M.I.T.'s differential analyzer advances science by freeing it from pick-and-shovel work of mathematics." The wishful student is brought to his senses when later he reads that, "the analyzer cannot solve problems the mathetician cannot do himself."

A careful examination of this pictorial description of problem solving will reveal to the student the necessity of a well planned procedure throughout every step. What does the mathematics classroom have in common with this ultra-modern mathematics laboratory? Three things are evident:

1. The problem situation must be recognized by the student;
2. The individual elements of any problem have their relation to each other even though that relationship be continually changing; and
3. An operative or manipulative procedure must be devised, consistent with known mathematical principles, through which the quantitative value of the unknown may be obtained and checked with a minimum of effort.

The mathematics classroom should be a workshop where the symbols and processes of mathematics are more than just a ritual and so often void of meaning. There are many problems that can be studied by the use of "multi-sensory aids." At every stage of solving a problem the student should be encouraged to use any device that will insure complete understanding. One of the simplest devices that can be placed in the hands of the student is a sheet of graph paper. A straight-edge will suffice to complete the machine, or call it a "mechanical analyzer" if you will.

The readers of this Journal will recall a number of recent articles illustrating the use of graphic forms in the solution of algebraic problems. The following material is presented to illustrate a simple treatment of some time-worn problems, which are well within the grasp of ninth year algebra students.

Let us suppose that the mathematics class is studying about mixture problems. If material (a) is mixed with material (b) then $a+b=c$, and if from (a) something is removed, the idea might be expressed by $a-b=c$. If all of the data concerning one of the initial mixtures is known it is evident that (n) is a part of (a) as well as part of (c). If x, y, and z are expressed as decimal parts of a, b, and c respectively, it follows that:

1. $xa=n=z(a+b)=zc$

or written

$$a/b=z/(x-z)$$

2. $xa=z(a-b)$

or written

$$a/b=z/(z-x)$$

3. $xa+yb=z(a+b)$

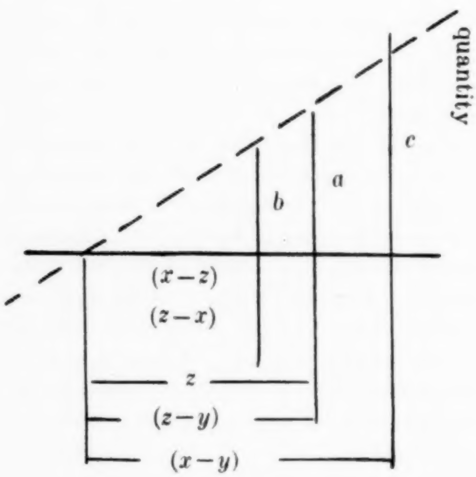
or written

$$a/b=(z-y)/(x-z)$$

The above equations when arranged in the form of a proportion lend themselves easily to a graphic interpretation. The four terms of the proportion can be represented on the legs of similar right triangles and the straight-edge forming the hypotenuse of the reference figure (See Figure 1). Any change in the numerical data may be

studied objectively by simply moving the straight-edge into a new position across the two scales.

The student will find many mixture problems in which all data concerning the final mixture are known. This condition may be represented as follows: $xa+y(c-a)=zc$, or written as a proportion $a/c=(z-y)/(x-y)$. The terms of this proportion are likewise found in the graphic representation of Figure 1.



Decimal parts

FIG. 1

There are a number of practical problems the solution of which lead to equations of the form: $1/a+1/b=1/c$ or $(a+b)/a=b/c$, and $1/a-1/b=1/c$ or $(b-a)/a=b/c$. These problems need not be enumerated to the teacher familiar with elementary physics or aeronautics, and can be found in almost any algebra book. This class of problems can be treated graphically as illustrated in Figures 2 and 3. Again the simple operation of shifting the position of the straight-edge makes possible the valuation of an unlimited number of problems.

The student might find it interesting to study the addition and subtraction of fractions by the graphic method (See Figure 4). To add or subtract fractions whose numerators are unity we observe

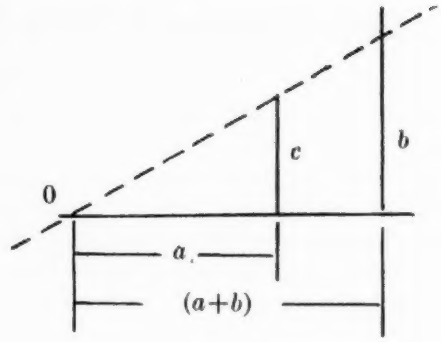


FIG. 2

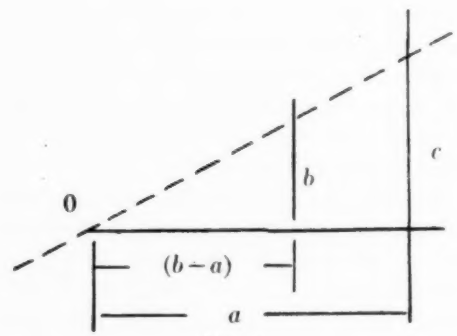


FIG. 3

that $1/a+1/b=1/c$ and $1/a-1/b=1/c$. Fractions of the form $n/a+m/b=s/c$ may be written

$$\frac{1}{a/n}+\frac{1}{b/m}=\frac{1}{c/s}$$

and thus conform to the equation $1/a+1/b=1/c$. If more than two fractions are to be added the position of the straight-edge must be shifted to accommodate each group of two addends until the total is found. Note that the straight-edge is first placed across scales a and b thus determining the value of c, and then shifted to the second position thus giving the value of $1/c$.

There are many problems which lead to the use of the general linear equations of the form $y=kx$ and $y=kx+c$. The student will find it worth while to study each of these by use of straight-edge and scale.

A problem about discounts might be studied as illustrated in Figure 5. If we

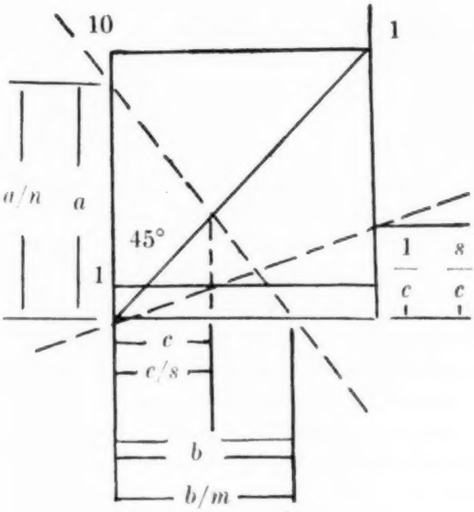


FIG. 4

start with the equation $(m-d)=c$ or $m-rm=c$, it follows that $r/1=(m-c)/m$ and also $m/1 \quad c/(1-r)$.

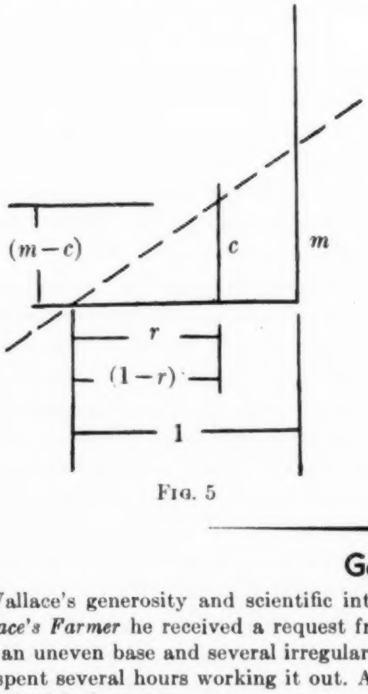


FIG. 5

Distance problems are likewise studied with a minimum of effort. The student starts with the relation $d=rt$ and thence to the equations $t/1=d/(r+r')$ and $r/r'=(t-k)/t$. The graphic layout is shown in Figure 6.

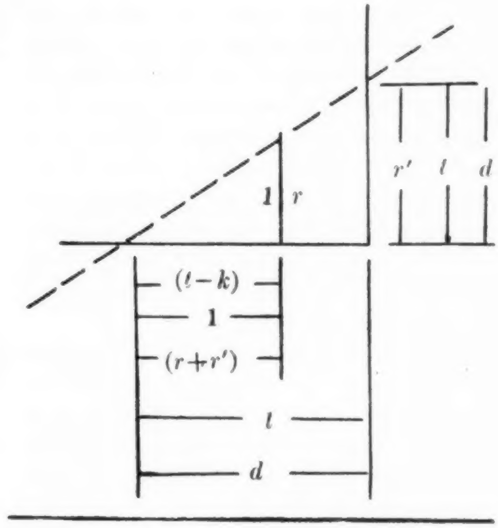


FIG. 6

The foregoing list of illustrations can be greatly extended. The teacher and student get only as much out of problem solving as they put into it. The time and effort given to the use of straight-edge and scale has its rewards. Most students enjoy the task of building the machine and seeing it work, not to mention the insight gained in problem solving.

Good Neighbor

Wallace's generosity and scientific interests are widely known. Once when he was editor of *Wallace's Farmer* he received a request from a reader for the content in cubic inches of a jar with an uneven base and several irregular protruding sections. He was fascinated by the problem and spent several hours working it out. A few days later he received a similar request and again obliged with the answer.

When a third such letter came in, Wallace investigated. He found that Iowa storekeepers were filling jars of odd shape with navy beans and offering prizes to those who could guess how many beans the jars held. Farmers had no trouble in counting the beans in a cubic inch, but when it came to figuring out how many cubic inches there were in the jars, they decided to "let Henry do it."

N.Y. Times Magazine, Sept. 15, 1946

On the Measurement of Angles*

By W. C. JANES
Manhattan, Kan.

IN SELECTING a topic for discussion on this program which deals with *Mathematics in Human Affairs* the speaker has chosen to talk about the subject of angular measure. In some of his former talks he has discussed such matters as how to figure silage, compute the amount of hay in a stack, and other subjects. Though the topic for today may not be intimately connected with problems in farm arithmetic, yet the speaker hopes that his audience will find it interesting, even though it may be of limited vocational value.

We all are familiar with the degree as a unit of angle in measurement, and we know that the degree represents an angle at the center of a circle whose sides intercept an arc which is 1/360th part of that circle. In other words, 360° make one revolution. The history of the degree goes back to very ancient times; and it probably was first defined by some ancient Babylonian shepherd who made use of his leisure time to study the motion of the stars. It appeared that there were 360 days (or thereabout) in a year. Consequently the degree was taken as the distance which the sun moves through the stars in one day. As the knowledge of astronomy progressed, and it became evident that a year contained about 365 days, one would suppose that the definition of the degree would have kept pace with the newer developments. But even after the astronomers realized that the number 360 did not accurately represent the number of days in a year, the degree remained as one-three-hundred-and-sixtieth part of a complete revolution. Why was this number retained? Possibly the fact that custom had established its use may have had some influence on its retention. But probably the real reason is somewhat different than that. As a matter of fact, 360 is a rather convenient number; for it is divisible by so many different factors. It is divisible by 2, 3, 4, 5, 6, 8, 9, 10, 12, 18, 20, and many other numbers. The fact that it can be broken up into fractional parts in so many different ways likely is the real reason why it was retained in the definition of the unit of angular measure.

But as the centuries passed, it began to appear that for certain purposes the degree was not the most suitable unit for the measurement of angles. Of course, the problem of angular measurement is intimately related to the properties of the circle; and two of the most prominent quantities connected with a circle are its diameter and its circumference. The ratio of the circumference to the diameter long remained an unsettled question. This ratio is the number commonly called *pi*, the value of which roughly is 3 1/7. But regardless of the value of *pi*, a convenient way to measure the size of a central angle in a circle is to express the ratio between the length of the intercepted arc and the length of the diameter. Or rather, since the radius of the circle figures prominently in the generation of an angle at its center, it is more convenient to give the size of this central angle by giving the ratio of the length of the intercepted arc to the length of the radius. This method of angular measure is what is known as *radian measure. A radian is an angle of such size that if its vertex be taken as the center of a circle, the intercepted arc will be the same length as the radius.* Radian measure is far more convenient than degree measure for many theoretical problems. The novice is not likely to believe this fact; for he knows that there are 2π radians in a revolution, and that π is incommensurable. However, the fact of the matter is that by the inherent nature of things, when one is making calculations related to the circle, π is likely

* **Radio talk over station KSAC.**

to enter into the problem. The radian is designed to take this fact into consideration, whereas the degree is not. Many formulas dependent upon degree measure contain the awkward factor $\pi/180$, while the corresponding formulas when expressed in radians do not involve this troublesome factor. Mathematicians have long realized the theoretical advantage of the radian over the degree. But the use of the radian as such is of rather recent origin; and the practical engineer has become sympathetic toward it only in very recent times.

But even the use of the radian does not lead to Utopia. The radian is rather a large angle. It contains about 57.3°. Consequently for many purposes a much smaller unit is preferable. Even the degree is too large for certain purposes. The artillery branch of the army seems to have hit upon a suitable unit of angular measure called the *mil.* The true mil is supposed to be one-onethousandth of a radian. Since there are 2π radians in a revolution, there are approximately 6283 true mils in a revolution. But 6283 is not a convenient number, for its prime factors are 61 and 103. Consequently the true mil presents its own difficulties as a suitable unit of angular measure. However, the mil has its advantages: for the practical meaning of it is *that an object one yard high will subtend an angle of one mil at the eye of the observer, provided the observer is 1000 yards away from the object.* To make this practical definition of the mil more precise, instead of using an object one yard high, one should use a circle arc one yard long which is part of the circle whose radius is 1000 yards. But a circle arc of length one yard, lying on a circle of one-thousand yard radius, is practically equivalent to a straight line one yard long. Consequently, the substitution of a straight line segment for a circle arc introduces only a slight error.

But let us return to the awkward number 6283. We have already stated that there are about 6283 true mils in a revolution. The unhandiness of 6283 is such that it seems to have led to a revision of the definition of the mil. The number 6283 is relatively near 6300. But 6300 is not nicely suited to desirable subdivision. The number 6400 is not very close to 6283. But 6400 can be subdivided into fractional parts with relative ease. The number 6400 is used by the artillery; and the army mil is defined as a central angle whose intercepted arc is one-sixtyfour-hundredth part of the circle. This definition still allows one to state that the army mil is such that *a length of one yard at a distance of 1000 yards subtends an angle of approximately one mil at the eye of the observer;* and this approximation does not introduce serious error unless the observed angle is rather large. However, when using the army mil in conjunction with the idea of "one yard at a thousand," careful accuracy requires the computation and use of what may be called a distortion factor. A discussion of this distortion factor would lead to considerations which are beside our purpose for today. We merely introduce this discussion of the mil to emphasize the fact that in certain types of practical problems it seems desirable to adopt a unit of angular measure other than the degree; and that in the choice of this unit, the theoretical advantages of the radian exert a distinct influence.

We must mention the fact that other methods of defining the unit of angular measure have been used. For example, there is a unit called the *grade,* which is *1/100th part of a right angle.* A person who is primarily interested in decimal calculation is likely to feel that the grade is superior to any other unit. However, this unit does not seem to have gained much popularity in the United States. It is the speaker's belief that the failure of the grade to become popular is due to the fact that its apparent advantage from the decimal point of view is more than offset by its failure to agree with desirable theoretical considerations.

The speaker is rather interested in the mil. A comparison of the mil with the de-

gree, shows that 1° is approximately equal to 18 mils, though 1° is a little short of 18 mils. This comparison is based on the definition of the army mil. If one compares what we have called the true mil with the degree, he finds that there are slightly more than 17 mils in a degree. It seems that at present the only place where the mil is extensively used is in the artillery branches of the armed services. But history has shown that frequently an idea which has first been used by the military authorities later is adopted into civilian use. Which way will the mil go? Will it become a popular civilian unit? Will it become less popular than at present? Or, will it remain much as it is in its present status? The answer to these questions will be determined by the future.

A New Method for the Solution of Cubic Equations

By Frank J. McMahon
Technical High School, Springfield, Mass.

Cubic equations can be solved by a method similar to that used in the extraction of a cube root.

This method has the following advantages over solution by the use of Horner's Method or Newton's correction formula:

(1) It is faster

(2) Being a straight arithmetic process it requires no preliminary graphing, approximation of location of roots, etc.

(3) It is easier to learn.

Consider the equation:

$$X^3+5X-12{,}282=0$$

by transposition

$$X^3+5X=12{,}282.$$

The numbers 5 and 12,282 are used in the extraction of the root as follows: The digits of 12,282 are blocked off in groups of three from the decimal point as in a cube root example: $\widehat{12}\ \widehat{282}$ and the largest number whose cube will go into the number 12, that is 2, is noted. (The 2 of course means 20.) The 20 is squared giving 400 to which is added the 5. Then 405 is multiplied by the 20 giving 8100 which is subtracted from 12,282 leaving 4182. A trial divisor is secured as in the cube root system by tripling the square of the present answer: $3\times400=1200$. Then: $1200+5=1205$. This is seen to go into 4182 three times, so the rest of the divisor is produced as follows: $3\times20\times3=180$ and 3 squared $=9$. The total divisor is 1394. Three times $1394=4182$ leaving no remainder so a root of the equation is 23.

The work is laid out in this manner:

$$\begin{array}{r} 400 \\ 5 \\ \hline 405 \\ 1200 \\ 5 \\ \hline 1205 \\ 180 \\ 9 \\ \hline 1394 \end{array} \qquad \begin{array}{|l} \multicolumn{1}{l}{\;\;2\;\;\;3} \\ \widehat{12}\widehat{282} \\ \;\;8100 \\ \hline \;\;4182 \\ \\ \\ \;\;4182 \\ \hline \end{array}$$

It can be seen that this process differs from the arithmetic extraction of cube root only in the addition of the coefficient of the X term to secure the trial divisor.

One root was secured in the above equation to show the general method. Now consider a more typical equation:

$$X^3+7X^2-7X-2=0.$$

If we let K equal the number used as the dividend in the process and M equal the number added in to secure the trial divisor it can be shown that in the equation:

$$ax^3+bx^2+cx+d=0$$

$$M=9ac-3b^2$$

and:

$$K=9abc-2b^3-27a^2d.$$

Since in the above equation:

$$a=1,\ b=7,\ c=-7,\ \text{and}\ d=-2:$$

$$M=-210\ \text{and}\ K=-1073.$$

The fact that M is negative means that in each case 210 is subtracted to secure the trial divisor, while K being negative simply means that the answer will be negative, but that 1073 is to be considered positive during the process.

With M a large number (absolute), and being negative it would seem to be very difficult to estimate the right number with which to begin the answer, but this is very simply taken care of by using the number whose cube is the smallest perfect cube above 210. If this number is too small the error will be made up in the next step after the trial divisor is found. In this equation the first number put in the answer should be 16, but if 15 is used as suggested above, a trial divisor 465 will be secured which will be seen to go into the remainder just once so the number 16 is found as $15+1$.

To secure each trial divisor it is necessary to find 3 times the square of the "present answer." This process can be simplified

(4)	(3)	(2)	(1)	15
				1
768	768	675	225	(−) 16.57515
48	−210	−210	−210	(−) 1073
0.75	558	465	15	225
816.75	24	45		848
−210.	0.25	1		511
606.75	582.25	511		337.000
3.465				291.125
0.0049				45.875000
610.2199				42.715393
				3.159607
				3.069715
				.089892
				.061420
				.028472

(5)	(6)
816.75	823.6947
6.930	.49710
0.0147	.000075
823.6947	824.191875
−210.	−210.
613.6947	614.191875
.24855	.0049725
.000025	.00000001
613.943275	614.19684751
	614.20

(7)	(8)
824.191875	824.20182003
.0099450	.004972530
.00000003	.0000000075
824.20182003	824.2067925675
−210	
614.20182003	
.002486265	
.0000000025	
614.2043062975	

considerably. The divisor 582.25 is secured in this equation as follows: $3\times16^2=768$. Subtracting 210 leaves the trial divisor 558 which is seen to go into 337 only 0.5 times. Then $3\times0.5\times16=24$, and 0.5 squared equals 0.25:

$$\begin{array}{r} 768 \\ -210 \\ \hline 558 \\ 24 \\ 0.25 \\ \hline 582.25 \end{array}$$

Now, in the next step, when it is necessary to find 3 times the square of 16.5, we simply use the three numbers checked:

$$1\times768+2\times24+3\times0.25=816.75.$$

Explanation:

$$3(16+0.5)^2$$
$$=3[16^2+2(16)(0.5)+0.5^2]$$
$$=1\times(3)16^2+2\times(3)(16)(0.5)+3(0.5)^2.$$

The process may also be shortened by the elimination of some decimal places in multiplying. The addition of each figure in the answer would ordinarily mean affixing 3 zeros to the dividend but experimentation has shown that an answer correct to five decimals can be secured using only six decimal places in the dividend.

Following is the work in securing the first root of the equation?

NOTE: (1) The last figure in the answer is seen by inspection to be closer to 5 than 4. It is not necessary to multiply in this case.

(2) Three times the square of the answer is found to be used in determining the other two roots.

Finding the other two roots:

In any cubic equation the sum of the products of the roots taken two at a time equals the coefficient of the first power term. (In this case "M"). Where there is no second power term the sum of any two roots equals the other root. Therefore if we call the root found by the above process "Z," and one of the other roots "Y," then the third root will equal $-Z-Y$.

The sum of their products taken two at a time is:

$$-Y^2+ZY-Z^2 \qquad -Y^2+ZY-Z^2=M$$

Solving:

$$Y=\frac{-Z\pm\sqrt{-4M-3Z^2}}{2}.$$

In our equation $3Z^2$ has already been found to ten decimals. This after combining with $-4M$ will yield a square root correct to five decimals: 3.97407, and then two roots of the equation: 10.27461, and 6.30054.

These, of course, are roots of the transformed equation. Those of the original equation may be found by the formula:

$$X=\frac{Z\ (\text{or } Y)-b}{3a}$$

which gives
the roots: -7.85838, -0.23315, 1.09154. Checking the results against coefficients in the original equation, we find the product of the roots to be 1.99990 and their sum -6.99999 as compared to $(-)2$ and -7 respectively.

Imaginary roots are found in the same way. The first equation considered yielded a real root: 23. The other roots are found to be:

$$\frac{-23\pm1\sqrt{1607}}{2}.$$

The author first learned to solve cubic equations by Horner's Method, and for a number of years taught it by the use of Newton's correction formula, but is convinced that the above method is faster and simpler than either. The only actual time studies that have been made, however, have been with him doing the calculating and therefore their validity may be questionable.

A Perpetual Calendar Formula

By David Skolnik
Central Commercial and Technical High School, Newark, N. J.

1. Given any date in the calendar, the following formula gives the day of the week on which it occurs:

$$D_w=[1\tfrac{1}{4}Y]-2C+[2\tfrac{1}{2}M]+D_m+\left[\frac{1}{|10-M|}\right]+\left[\frac{3}{M+1}\right]+\left[\frac{3L}{M+1}\right]$$

where

$D_w \equiv$ day of the week (mod 7); for example, $D_w=9\equiv 2=$ Monday

$Y=$ year in any one century; for 1939, $Y=39$

$C=$ centurial year 16, 17, 18, or 19; for 1939, $C=19$

$M=$ month; for January, $M=1$

$D_m=$ day of the month; for February 20, $D_m=20$

$L=0$ for leap years; $L=1$ for all other years.

Note 1. $[x]$ means the greatest integer less than or equal to x; for example, $[2\tfrac{1}{2}]=2$, $[3]=3$, $[1/3]=0$.

Note 2. In the fraction

$$\frac{1}{|10-M|}$$

use the absolute value of $10-M$. If $M=10$, the fraction has no value—drop it.

Note 3. For $C=20$, 24, 12, 8, $\cdots$, use $C=16$; for $C=21$, 25, 13, 9, $\cdots$, use $C=17$; and so on. The calendar repeats itself every 400 years.

Note 4. Every leap year is divisible by 4. However, if the year ends in 00 it is a leap year only if the centurial year itself is divisible by 4. Leap years: 1924, 1872, 2000, 2400, $\cdots$; non-leap years: 1938, 2050, 1900, 2100, $\cdots$.

Note 5. For dates prior to September 14, 1752, add 11 (or 4) to the final answer for D_w. The calendar was changed in the United States and in England on that date.

Example 1. On what day of the week did November 11, 1918 (the World War I armistice) fall?

Here $Y=18$, $C=19$, $M=11$, $D_m=11$ $L=1$. Hence

$$\begin{aligned}
D_w&=[1\tfrac{1}{4}\cdot 18]-2\cdot 19+[2\tfrac{1}{2}\cdot 11]+11+\left[\frac{1}{|10-11|}\right]+\left[\frac{3}{11+1}\right]+\left[\frac{3\cdot 1}{11+1}\right]\\
&=[22\tfrac{1}{2}]-38+[27\tfrac{1}{2}]+11+\left[\frac{1}{1}\right]+\left[\frac{1}{4}\right]+\left[\frac{1}{4}\right]\\
&=22-38+27+11+1+0+0\\
&=23\equiv 2=\text{Monday}
\end{aligned}$$

Example 2. On what day of the week will the President of the U. S. be inaugurated in 2097, i.e., Jan. 20, 2097?

Here $Y=97$, $C=16$ (See Note 3), $M=1$, $D_m=20$, $L=1$. Hence

$$\begin{aligned}
D_w&=[1\tfrac{1}{4}\cdot 97]-2\cdot 16+[2\tfrac{1}{2}\cdot 1]+20+\left[\frac{1}{|10-1|}\right]+\left[\frac{3}{1+1}\right]+\left[\frac{3\cdot 1}{1+1}\right]\\
&=[121\tfrac{1}{4}]-32+[2\tfrac{1}{2}]+20+\left[\frac{1}{9}\right]+[1\tfrac{1}{2}]+[1\tfrac{1}{2}]\\
&=121-32+2+20+0+1+1\\
&=113\equiv 1=\text{Sunday.}
\end{aligned}$$

Example 3. On what day of the week will the following Presidential inaugural occur, i.e., Jan. 20, 2101?

Here $Y=01$, $C=17$, $M=1$, $D_m=20$, $L=1$. Hence

$$D_w=[1\tfrac{1}{4}\cdot 1]-2\cdot 17+[2\tfrac{1}{2}\cdot 1]+20+\left[\frac{1}{|10-1|}\right]+\left[\frac{3}{1+1}\right]+\left[\frac{3\cdot 1}{1+1}\right]$$
$$=\ 1\ -34+\ 2\ +20+\ 0\ +\ 1\ +\ 1$$
$$=-9\equiv 5=\text{Thursday}.$$

2. The twentieth amendment to the Constitution of the U. S. sets January 20 as the date for inaugurating the President. It has been said that this date is less likely to fall on Sunday than some other dates that might have been chosen. It is interesting to examine this with the perpetual calendar formula.

Every inaugural year is 4 years later than the preceding one. Therefore, the term $1\frac{1}{4}Y$ in our formula increases by 5 for two successive inaugurals that are in the same century. In other words, within any one century, D_w follows the series

$$1, 6, 4, 2, 7, 5, 3, 1, 6, 4, 2, 7, 5, 3, 1, \cdots.$$

As may be verified with the formula, the difference 5 also holds between the inaugurals of 1997 and 2001. However, in making the other three crossings between the centuries of the calendar cycle, the difference is only 4. Examples 2 and 3 above show that Jan. 20, 2097 is on Sunday, and Jan. 20, 2101 is on Thursday. This means that in our series D_w jumps from 1 to 5, thus skipping 6, 4, 2, and 7.

Again, for Jan. 20, 2197, $D_w=6$; and for Jan. 20, 2201, $D_w=3$. This skips 4, 2, 7, and 5.

Finally, for Jan. 20, 2297, $D_w=4$; and for Jan. 20, 2301, $D_w=1$. This skips 2, 7, 5, and 3.

It can now be seen that in the 400-year calendar cycle, every day except Sunday is omitted from its regular turn one or more times because of the change in the centurial year. In other words, the inaugural date January 20 will fall on Sunday more often than any other day of the week—4 times in each century, or 16 times in the four-century cycle of the calendar. Since Saturday and Monday are omitted 3 times by the changes in the centurial year, January 20 falls only 13 times on either of these days. This means that Jan. 19 or 21 as an inaugural date would fall on Sunday less often than any other day of the week.

Teaching Logarithms for Use

Many algebra textbooks seem to use logarithms chiefly as a method of teaching interpolation. As a result, students get the false idea that logarithms are hard to use. Engineers and others who really use logarithms always work with tables that have proportional parts. Each of our students is given such a table, printed on a single sheet. Most students quickly become quite proficient in the use of this table. At a cost of about three cents per student, logarithms become useful tools, available for all further computation.

—W. C. Bartlett, Watertown, in the *Connecticut Teacher* for January 1946

A Copy Book for Arithmetic*

By Clyde H. Lady
State Teachers College, Slippery Rock, Pa.

One day early in 1810 Benjamin Harris went to Stewart's Store in Menallen Township, Adams County, Pennsylvania, to buy himself a blank book for the study of arithmetic. The book, extant and in the possession of the writer's family, is composed of twenty-five sheets of plain white paper, thirteen by eight inches, bound between two pieces of heavy wrapping paper. The binding is merely a strong thread whipped around and around the one edge.

Judging from the dates of entries, one concludes that Benjamin was in this school, almost certainly a private school, about two months during February and March. He must have been one of the older boys, for the problems deal with proportion, interest and discounts, rebates and factor's commissions, compound fellowship, the double rule of three, duodecimals and other topics. A problem is stated, then its solution follows. Naturally there were portions of a page not filled with work. Many of these Benjamin marked off and filled in with other material. In one corner he gives a much earlier version of the "Geometric Spelling" of TOBACCO to which reference is made in the January issue, page 23. Benjamin's version follows.

Take three-fourths of a cross and a circle compleat
With an upright where two simme circles meet
With angle tryangle a-standing on feet
With two simme circles and a circle compleat.

Here are several other fillers-in:

"Benjamin Harris, his hand and pen;
He may be good, but who knows when?"

"Trust today and pay tomorrow,
And that will keep us all from sorrow."

"The double rule of three does puzzle me."

Here also are some of the problems; they show how greatly the subject matter of that period was influenced by English practice.

How many barleycorns will reach around the globe of the earth which is 360 degrees and each degree sixty-nine miles and a half?

How many times doth a wheel which is 18 feet 6 inches round turn between London and York which is 150 miles?

If a gallon of ale costs 8d. what will 36 gallons cost? The next question states that a gallon of beer costs 4d.

At a noble per week how many weeks board may I have for 50?

If 15 shillings worth of wine will serve 46 men when a tun is worth 12, how many men will the same 15 shillings worth suffice when the tun is worth but 9?

Suppose I allow my correspondent 1¾% for provision, what may he demand on the disbursement of 70 15s 4d?

A draper bought 100 yards of broadcloth for 56; I demand how he must sell it per yard to gain 15 in laying out 100.

Three merchants trading to Virginia lost goods to the amount of 800. Now if *D*'s stock was 1200, *E*'s 4800 and *F*'s 2000 what sum did each man lose?

A and *B* found a bag of money and disputed who should have it. *A* said the half, third and fourth of the money made 130, and if *B* could tell how much was in the bag he should have all of it; otherwise he should have nothing. I demand how much was in the bag. This problem was not worked by algebra, as would be done today, but by proportion. This next one comes under "Double Position" (see any History of Mathematics). *A*, *B* and *C* would divide 100 between them so that *B* may have 3 more than *A*, and *C* 4 more than *B*. I demand how much each man shall have.

* These comments were formulated after reading the few lines on "Geometric Spelling" on page 23 of the January 1946 issue of *The Mathematics Teacher*.—Author.

A Problem in Elementary Algebra

By Roy Dubisch
Montana State University, Missoula, Montana

The following problem, while not new, seems to have escaped the attention of text-book writers in the particular form given here and hence it is perhaps unknown to many teachers. Its value lies not only in the keen interest it stimulates in students in this aeronautical age but in it being an example of the use of algebra in correcting "common-sense" judgments.

Suppose a plane has a cruising speed of 200 miles an hour and a 4 hour fuel supply. This means that, without any wind, it can fly 800 miles before running out of fuel. If, then, the pilot wishes to fly out to sea and back again, he can, without wind, fly 400 miles before turning back. Now suppose he is flying out with a tail wind of 20 mph so that his actual (ground speed) going out is 220 mph. Coming back he flies against a head wind of 20 mph so that his actual speed is 180 mph. How far out can he fly and return?

Fully 90% of my students answer 400 miles (the others hesitate—probably because they suspect a trick)! The algebra is simple, of course. If d is the distance out (and back), $d/220$ is the time out; $d/180$ is the time back. Hence $d/220+d/180=4$ so that $d=396$. However, it is desirable to first point out the fact that the time spent in flying 180 miles is more than that spent in flying 220 miles so that the average speed is not 200 miles and to comment that the same "logic" would apply to a 200 mph wind (ignoring the aerodynamics!)—in which case he would not return at all.

The reaction of students to this problem is always interesting and sometimes illuminating. One student said "By science I know that he can't fly out as far but by algebra it comes out the same"—an all too typical attitude towards algebra! But perhaps the most amusing retort was from an air-corp student in the C.T.D. here who was positive that he couldn't fly out as far with the wind as without the wind. When asked on what basis he formed his judgement he laconically replied, "I tried it"! (over land however—landing in a meadow instead of at the airport).

This problem is, of course, a special case of the "radius of action" problem which, when the wind is not a head wind, is usually solved by graphical methods. It illustrates the aeronautical adage "on a round trip, a wind always hinders."

National Council Yearbooks!

The National Council Yearbooks are rapidly going out of print. The October (1946) issue contained a list of those still available and the prices for which they may be obtained. Teachers who wish complete files and particularly school libraries who lack certain books should order now.—Editor.

Henry Holt & Company, Publishers, 257 Fourth Avenue, New York, N. Y., have printed copies about 14×20 inches on firm, but not stiff paper, of "The River Mathematics," which appeared as the frontispiece in the October (1945) issue of The Mathematics Teacher.

Anyone wishing copies of this picture may secure the same by sending 10¢ to cover the mailing tubes and postage cost, to the above address.—Editor.

◆ THE ART OF TEACHING ◆

The General Method for Solving a Quadratic Equation[1]

By A. E. Mallory
Colorado State College of Education, Greeley, Colorado

Instructor: In an algebraic expression such as $y=2x^2+3x-1^2$ what are the two variables?

Gale: x and y. y is the dependent variable.

Instructor: When the highest power involved in such an expression is a square, you will recall that we defined such a function as a "quadratic function."

Instructor: In our graphic illustrations of these quadratic functions, we found that the graphs may or may not cut the x-axis in two points as shown in our text. If the graph of the quadratic under discussion cuts the x-axis, y, or the function itself, is zero. What do the abcissas or x distances of these points of intersection become?

Answer: Roots of the equation.

Instructor: Finding the roots of an equation is termed solving the equation. Then what must the expression, "solving the equation," mean, John?

John: Finding the values of x for which y equals zero.

Instructor: How many methods are there which we can use in solving a quadratic equation?

Fred: Graphic.

Another student: Factoring. Setting each factor equal to zero and solving.

Instructor: A third method?

Gale: Completing the square

Francis: General method.

Instructor: Our work today then will be to develop this general method for solving a quadratic equation. The third method, completing the square, will serve only as a step in the development of the general method for solving a quadratic equation. Let us start with this equation: $y=5x^2+8x+3$. We shall solve it first by which one of the above methods?

Answer: Completing the square.

Instructor: Note carefully the steps in completing the square.

Robert: Set y equal to zero; then $5x^2+8x+3=0$.

Instructor: Second step, Clarence.

Clarence: Factoring.

Instructor: Fred, second step.

Fred: Subtract 3 from both sides: $5x^2+8x=-3$.

Instructor: Next step now, Gale.

Gale: Complete the square.

Francis: Divide each term by the coefficient of x^2, giving

$$x^2+\frac{8}{5}x=-\frac{3}{5}\,.$$

Instructor: Fourth step, John.

John: Put in the third part of the equation. Square that. Take half of the coefficient of x and then square it. Add it to both sides.

Robert: That will give us

$$x^2+\frac{8}{5}x+\left(\frac{8}{10}\right)^2=-\frac{3}{5}+\frac{64}{100}\,.$$

[1] This demonstration was conducted by Mr. Mallory in the intermediate algebra class in the training school of the Colorado State College of Education.

[2] This equation together with all equations and algebraic expressions involved in this discussion were written on the blackboard as the development proceeded.

Instructor: What led you to make that answer, Robert?

Robert: Well, I found out before that the third term was always half of the coefficient of the second term squared.

Instructor: Now the fifth step will be to simplify the resulting equation.

Eleanor: The expression

$$\left(x+\frac{8}{10}\right)^2=\frac{-60+64}{100}=\frac{4}{100}\cdot$$

Instructor: See what's happened here. The right hand member has been added algebraically by reducing to a common denominator, 100, and the addition performed. Now the sixth step.

Erhardt: Get your plus or minus in their squares by dividing.

Gale: Take the square root of each side.

Francis:

$$x+\frac{8}{10}=+\quad \text{or} \quad -$$

Clarence:

$$+\quad \text{or} \quad -\quad \frac{2}{10}\cdot$$

Instructor: Then we have (using black board)

$$x+\frac{8}{10}=\pm\frac{2}{10}\cdot$$

Instructor: Now solving this linear equation for one of the roots, using the plus sign, let us put it this way (illustrating on blackboard): $x_1=?$

Fred: Two-tenths.

John: x_1 would be $-\frac{6}{10}\cdot$

Instructor: Using the minus sign, x_2 equals -1.

Instructor: Now express the above equation in its general form as explained in our text. What will that be, Fred?

Fred: I don't know.

Instructor: Gale, what will this be in general form?

Gale: It will be the same thing, won't it?

Francis: y equals ax^2+bx+c.

Instructor: a takes the place of the coefficient of x^2; b takes the place of the coefficient of x; c takes the place of 3.

Robert: Now all you got to do is to substitute.

Instructor: Very well, but let us solve this general quadratic by completing the square, so that we can see what position these coefficients take in the final solution. What is the first step?

Answer: Set y equal to zero, or $ax^2+bx+c=0$ (written by instructor).

Instructor: Second step.

Answer: Subtract c from each side. $ax^2+bx=-c$.

Instructor: Third step.

Answer: Divide both sides of the equation by a.

$$x^2+\frac{b}{a}x=-\frac{c}{a}\cdot$$

Instructor: What are you going to divide by a for?

Answer: To remove coefficient of x^2.

Answer: To make a perfect square.

Instructor: No, you can make a perfect square without doing that.

Answer: To factor.

Answer: To find the square root.

Instructor: No, you have not yet sensed the real reason.

John: So you can get what you are after.

Instructor: What are we after? There's a real reason for dividing by a.

Eleanor: You tell us.

Instructor: (After the explanation of the unity and Hindu methods of completing the square of a quadratic trinomial): We are using the unity method. We divide by the coefficient of x^2 in order to make the coefficient one. We would rather have the fractions, which result from this process, than to have to remember the procedure involved in the Hindu method. Now the fourth step: Let us complete the square by use of information that is already in hand.

Fred: Can you keep the square over here on the left hand member?

Instructor: Can you, Gale?

Gale: No.

Instructor: What, then, completes the square?

Answer:

$$\left(\frac{b}{2a}\right)^2.$$

Instructor: Square it and add it to both sides of the equation, Gale.

Gale:

$$x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}-\frac{c}{a}.$$

Instructor: Now, John, simplify this equation. You have an addition to perform over here (indicating right hand member).

John:

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}.$$

Instructor: Where are your perfect squares now?

Eleanor: There is one on the left hand side and the denominator on the right hand side.

Instructor: Now we are ready to make use of these perfect squares. What axiom enables us to proceed?

Eleanor: If you take the square root of one side, you must take the square root of the other side.

Instructor: (After several attempts to take the square root of b^2-4ac): Do you know any binomials that are perfect squares?

Fred: Take the square root of all of it with the radical.

Instructor: (Illustrating on blackboard) Then we will have

$$x+\frac{b}{2a}=\pm\frac{\sqrt{b^2-4ac}}{2a}.$$

Instructor: Using the plus sign, what is x_1?

Gale:

$$x_1=\frac{-b+\sqrt{b^2-4ac}}{2a}.$$

Instructor: Using the minus sign, what is x_2, Erhardt?

Erhardt:

$$x_2=\frac{-b-\sqrt{b^2-4ac}}{2a}.$$

Robert: So that's where that formula came from.

Instructor: Now each root is stated in what terms of the general quadratic equation?

Eleanor: Coefficient.

Sam: Why, it's the plural, coefficients.

Instructor: What can you say as to the importance of the coefficients as a result of the preceding development?

Answer: The roots are in terms of the coefficients of the quadratic.

Instructor: Since the roots of the general quadratic are expressed in terms of the coefficients of the quadratic, we shall use this solution as a formula for solving quadratic equations.

EDITORIALS

THE NEW EDITOR OF THE AMERICAN MATHEMATICAL MONTHLY

BEGINNING with the January (1947) issue, Professor C. V. Newsom of Oberlin College, Oberlin, Ohio, will be the editor-in-chief of *The American Mathematical Monthly*. Professor Newsom has been an associate editor since 1943. During World War II he had charge of the department of *War Information*, and since the war it has been called the department of *General Information*.

Professor Newsom started his career as a high school teacher and at one time was head of one of the larger high school mathematics departments in the state of Kansas. Later as Head of the Mathematics Department at the University of New Mexico, he organized the Southwestern Section of the Mathematical Association in the New Mexico Educational Association. He has been at Oberlin now for three years.

Professor Newsom is very much interested in secondary school mathematics and attends the annual meetings of the National Council of Teachers of Mathematics. It is particularly fitting that he should be the editor now when the Association and the Council are cooperating so well for the improvement of mathematical education. THE MATHEMATICS TEACHER wishes Professor Newsom the greatest success in his new work. W. D. R.

THE AUSTRALIAN MATHEMATICS TEACHER

THE MATHEMATICS TEACHER is glad to welcome a new magazine in the field of mathematics—*The Australian Mathematics Teacher*. The editor-in-chief is Professor I. S. Turner of Teachers College, Sydney, N. S. W. The July (1946) number was the second of Volume 2.

Professor Turner did a great deal of graduate work in mathematics at Cambridge University and then enrolled at Teachers College, Columbia University, where he received his Ph.D. degree in mathematics. He was the author of The Fourteenth Yearbook of the National Council of Teachers of Mathematics, *The Training of Mathematics Teachers.*

THE MATHEMATICS TEACHER wishes to take this opportunity to wish Professor Turner the greatest success in his new venture. W. D. R.

IN UNION THERE IS STRENGTH

THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS is the only country-wide organization devoted to the interests of elementary and secondary school teachers of mathematics. THE MATHEMATICS TEACHER is trying to help teachers in these fields by publishing articles of varying degrees of difficulty and fields of interest. We do not expect any one issue to please or help all readers. However, we hope that over a period of time each member of the Council will be satisfied. Now is the time for us to renew our group consciousness, purpose, loyalty and courage, and press on to better things for teachers. If all teachers of mathematics in the country were enrolled as members, our power and influence would be noticeably increased. Let us all endeavor to get new members in 1947. W. D. R.

◆ IN OTHER PERIODICALS ◆

By Nathan Lazar
Midwood High School, Brooklyn 10, New York

The Mathematical Gazette
July 1946, Vol. 30, No. 290.

1. Vajda, S., "Generalized Metrical Theorems," pp. 122–125.
2. Jaeger, J. C., "On the Behaviour of the Roots of an Algebraic Equation as the Coefficients Vary," pp. 126–128.
3. Lightfoot, N. M. H., "Motion of a Particle," pp. 129–131.
4. Picken, D. K., "The Logarithmic Function; and the Numbers e and π," pp. 132–136.
5. Eliezer, C. Jayaratnam, "On the Constrained Motion of a Rigid Body," pp. 137–140.
6. Camier, E. D., "A Property Characteristic of Quadrics of Revolution and General Cylinders," pp. 141–143.
7. Mathematical Notes, pp. 144–172.
8. Reviews, pp. 175–184.

School Science and Mathematics
November 1946, Vol. 46, No. 8.

1. Jerbert, A. R., "The Art of Summation," pp. 701–707.
2. Georges, J. S., "Teaching Functional Thinking in Mathematics," pp. 733–748.
3. Hartung, Maurice L., "The Order of Operations in Elementary Mathematics," pp. 752–754.
4. Problem Department, pp. 782–786.
5. Book Reviews, pp. 788–791.

Miscellaneous.

1. Beck, E. J., "Arithmetic Story Problems," *Instructor*, 55: 28, September 1946.
2. "Easy Addition Facts," *Instructor*, 55: 37, September 1946.
3. Guiler, W. S., "Difficulties in Percentage Encountered by Ninth-Grade Pupils," *Elementary School Journal*, 46: 563–573, June 1946.
4. Guiler, W. S., "Difficulties Encountered by College Freshmen in Decimals," *Journal of Educational Research*, 40: 1–13, September 1946.
5. Henderson, K. B., "Weaknesses in Arithmetic Teaching," *Elementary School Journal*, 46: 579–581, June 1946.
6. Holzinger, K. J., and Swineford, F., "Relation of Two Bi-factors to Achievement in Geometry and Other Subjects," *Journal of Educational Psychology*, 37: 257–265, May 1946.
7. Karstens, H., "Checking the Estimate in Long Division," *Journal of Educational Research*, 40: 52–56, September 1946.
8. Morton, J. A., "A Study of Children's Mathematical Interest Questions as a Clue to Grade Placement of Arithmetic Topics," *Journal of Educational Psychology*, 37: 293–315, May 1946.
9. Parry, M. E., "Ready-to-use Tests for Middle and Upper Grades: Problem Solving Processes," *Instructor*, 55: 39, October 1946.
10. Poquet, L., "Problem Analysis," *Instructor*, 55: 30, September 1946.
11. Rice, R., "Your Arithmetic Problems; How to Make Them Count," *Grade Teacher*, 64: 50+, September 1946.
12. Salit, C. R., "Geometry as a Natural Way of Thinking," *Education*, 67: 28–33, September 1946.
13. Steel, C. R., "Standardization in Arithmetic," *Journal of Education* (London), 78: 494+, September 1946.
14. Welch, P., "Practical Number Work," *Grade Teacher*, 64: 36+, October 1946.
15. Wiancko, F. H., "Mathematics Tuned to the Present Times," *School* (Secondary Edition), 35: 40–43, October 1946.
16. Wilson, M. B., "Arithmetic Comes Alive," *Instructor*, 55: 36+, October 1946.

Program of the Annual Meeting of the National Council of Teachers of Mathematics

Atlantic City, New Jersey
Chalfonte-Haddon Hall
February 28 and March 1, 1947

THURSDAY, FEBRUARY 27

8:00 P.M. Room A. Meeting of the Board of Directors of the National Council of Teachers of Mathematics.

FRIDAY, FEBRUARY 28

1. 10:30 A.M. to 12:00 Noon. Mandarin Room. Elementary School Section.
Presiding: Vera Sanford, State Teachers College, Oneonta, N. Y.
1.1 *Beginnings of Quantitative Generalizations.* Jane Plenty, State Teachers College, Newark, N. J.
1.2 *Teaching Methods of Studying Arithmetic.* H. G. Wheat, West Virginia University, Morgantown, W. Va.
1.3 *Discussion.* William Brownell, Duke University, Durham, N.C., Dennis Cooke, East Carolina Teachers College, Greenville, N. C., and Robert L. Morton, Ohio University, Athens, Ohio.
2. 10:30 A.M. to 12:00 Noon. Benjamin West Room. Senior High School Section.
Presiding: W. D. Reeve, Teachers College, Columbia University, New York City.
2.1 *A Classroom Teacher Looks at Textbooks.* Martha Hildebrandt, Proviso Township High School, Maywood, Ill.
2.2 *An Evaluation of Research Relating to Mathematical Education.* M. L. Hartung, University High School, Chicago, Ill.
2.3 *Mathematics for Responsible Citizenship.* Harold P. Fawcett, School of Education, Ohio State University, Columbus, Ohio.
2.4 *Discussion.*
3. 10:30 A.M. to 12:00 Noon. Tower Rooms 1333–1335–1337. Junior College Section.
Presiding: Albert E. Meder, Jr., Rutgers University, New Brunswick, N. J.
3.1 *Discussion. Junior College Problems.* Ellis R. Ott, Rutgers University, New Brunswick, N. J., Clair W. Black, Fairleigh Dickinson Junior College, Rutherford, N. J., and H. W. Brinkman, Swarthmore College, Swarthmore, Pa.
4. 12:00 Noon. *Luncheon for Delegates and State Representatives.*
5. 1:30–4:00 P.M. Benjamin West Room. Junior High School Section.
Presiding: Hubert B Risinger, East Orange, N. J.
5.1 *What Should We Teach and How Should We Teach It for Consumer Competence?* Lucien B. Kinney, Stanford University, Stanford, Calif.
5.2 *What Should We Teach and How Should We Teach It for Vocational Competence?* Walter H. Carnahan, Purdue University, Lafayette, Ind.
5.3 *Discussion.*
6. 1:30–4:00 P.M. Mandarin Room. Teacher Training Section.
Presiding: Virgil S. Mallory, State Teachers College, Montclair, N. J.
6.1 *Training of Teachers of General Mathematics.* H. E. Grime, Public Schools, Cleveland, Ohio.
6.2 *On-the-Job Training of Junior-Senior High School Teachers.* Rolland R. Smith, Public Schools, Springfield, Mass.
6.3 *The Training of Mathematics Teachers for the High School—Recommendations of the Cooperative Committee of the A.A.A.S.* E. H. C. Hildebrandt, Northwestern University, Evanston, Ill.
6.4 *Discussion*
7. 1:30–4:00 P.M. Sun Porch. Guidance Section.
Presiding: Lela A. Lynam, Wilmington High School, Wilmington, Del.
7.1 *Recruiting and the Economic Sta-*

tus of the Mathematics Teachers. Raleigh Schorling, University High School, Ann Arbor, Mich.

7.2 *Directed Segments of the School Population.* Veryl Schult, Washington, D. C.

8. 4:00–5:00 P.M. Tea. Garden Room.
9. 8:00–10:00 P.M. Vernon Room. General Meeting.

Presiding: Lucien B. Kinney, Stanford University, Stanford, Calif.

9.1 *Address of Welcome.* Madeline D. Messer, President Association of Mathematics Teachers of New Jersey.

9.2 *The Use of Celestial Navigation in Teaching Solid Geometry and Spherical Trigonometry.* John Reckzeh, State Teachers College, Jersey City, N. J.

9.3 *The Invalidity of the Indirect Method of Proof in Geometry Textbooks.* Nathan Lazar, Midwood High School, Brooklyn, N. Y.

9.4 *Some Developments in Twelfth Grade Arithmetic.* Ralph Miller, Ridgewood High School, Ridgewood, N. J.

9.5 *Discussion.*

SATURDAY, MARCH 1

1. 8:30–9:30 A.M. Business Meeting.
2. 10:00 A.M.–12:00 Noon. Garden Room. Junior High School Section.

Presiding: Hubert B. Risinger, Davey Junior High School, East Orange, N. J.

2.1 *A Look At The Future.* Hope Tisdale Eldridge, Population Division, Bureau of the Census, Washingon, D. C.

2.2 *Curriculum Experiments and Revisions:* Agnes Herbert, Baltimore, Md., Chairman, Mary A. Potter, Racine, Wis. Mary C. Rogers, Westfield, N. J. Ruth G. Sumner, Oakland, Calif. Edith Woolsey, Minneapolis, Minn.

2.3 *Discussion.*

3. 10:00 A.M.–12:00 Noon. Benjamin West Room. Senior High School Section.

Presiding: H. C. Christofferson, Miami University, Oxford, Ohio.

3.1 *Functional Thinking in Secondary Mathematics.* Mae Russell, State Teachers College, Salisbury, Md.

3.2 *Dynamic Beauty in Geometry—Slides and Demonstrations.* H. v. Baravalle, Adelphi College, Garden City, N. J.

3.3 *Mathematics an Essential of Culture.* Walter H. Carnahan, Purdue University, Lafayette, Ind.

3.4 *Discussion.*

4. 10:00 A.M.–12:00 Noon. Mandarin Room. Teacher Training Section.

Presiding: Foster E. Grossnickle, State Teachers College, Jersey City, N. J.

4.1 *A Usable Philosophy in Teaching Arithmetic.* Dennis H. Cooke, President, East Carolina Teachers College, Greenville, N. C.

4.2 *Preliminary Report of the M.A.A. on Training of Arithmetic Teachers in Ohio.* Harold P. Fawcett, School of Education, Ohio State University, Columbus, Ohio.

4.3 Third Speaker to be Announced Later.

4.4 *Discussion.*

5. 1:30–4:00 P.M. Garden Room. Elementary School Section.

Presiding: Holmes Boynton, State Teachers College, New Haven, Conn.

5.1 *Meaningful Arithmetic is Making Headway.* C. L. Thiele, Public Schools, Detroit, Mich.

5.2 *The Measurement of Understanding in Elementary School Mathematics.* Ben A. Sueltz, State Teachers College, Cortland, N. J.

5.3 *The Abacometer.* Nathan Lazar, Midwood High School, Brooklyn, N. Y.

5.4 *Discussion.* Harold P. Fawcett, Ohio State University, Columbus, Ohio. Amanda Loughren, Public Schools, Elizabeth, N. J. Freeman Macomber, Drake University, Des Moines, Ia.

6. 1:30–3:30 P.M. Benjamin West Room. Media of Instruction Section.

Presiding: E. H. C. Hildebrandt, Northwestern University, Evanston, Ill.

6.1 *Mechanism and Mathematics,* W. W. Rankin, Duke University, Durham, N. C.

6.2 *The Place of Multi-Sensory Aids in the Teacher Training Program.* Howard F. Fehr, State Teachers College, Montclair, N. J.

7. 1:30–3:30 P.M. Tower Rooms 1333–1335–1337. Film Section.
Presiding: Fred Bedford, State Teachers College, Jersey City, N. J.
7.1 *Showing of the Latest Films for Teaching of Mathematics.* Henry W. Syer, Boston University, Boston, Mass.
8. 4:00–5:00 P.M. Bakewell Room. Tea for Board of Directors.
9. 7:00 P.M. Rutland Room. Annual Discussion Banquet.
Toastmaster: W. D. Reeve, Teachers College, Columbia University, New York City.
Guest Speaker: C. V. Newsom, Oberlin College, Oberlin, Ohio.
Topic: A Philosophy for the Mathematics Teacher.